Validation of an isoparametric, 8-node EAS brick element in Chrono::FEA

Bryan Peterson
Antonio Recuero

June 25, 2016

Abstract This report describes the validation of Chrono's eight-node brick element with enhanced assumed strain (EAS). This element is a general-purpose three-dimensional element that can be employed to model a wide variety of material constitutive behavior. Currently, the users can select two different constitutive behaviors: Linear isotropic and hyperelastic Mooney-Rivlin. Both formulations are validated in this document agains ANSYS.

Keywords: 8-node brick element, enhanced assumed strain, Mooney-Rivlin material, Chrono

Contents

1. Introduction

Chrono's users may find a more detailed descriptions of the Chrono's brick element on the website: http://api.chrono.projectchrono.org/whitepaper_root.html.

2. Eight-noded brick element with Enhanced Assumed Strain

Chronofeatures an eight-noded brick element that can be used for the dynamic simulation of large deformation systems. This element has tri-linear shape functions and implements an Enhanced Assumed Strain (EAS) technique to circumvent locking. The nodal coordinates of the EAS brick are the position of the nodes, i.e. $\mathbf{r}^i(x^i, y^i, z^i) = \mathbf{S}^i(x^i, y^i, z^i)\mathbf{e}^i$, where the shape functions are given as follows

$$\begin{aligned} S_1^i &= \frac{1}{4}(1-\xi^i)(1-\eta^i)(1-\zeta^i), & S_2^i &= \frac{1}{4}(1+\xi^i)(1-\eta^i)(1-\zeta^i), \\ S_3^i &= \frac{1}{4}(1-\xi^i)(1+\eta^i)(1-\zeta^i), & S_4^i &= \frac{1}{4}(1+\xi^i)(1+\eta^i)(1-\zeta^i), \\ S_5^i &= \frac{1}{4}(1-\xi^i)(1-\eta^i)(1+\zeta^i), & S_6^i &= \frac{1}{4}(1+\xi^i)(1-\eta^i)(1+\zeta^i), \\ S_7^i &= \frac{1}{4}(1-\xi^i)(1+\eta^i)(1+\zeta^i), \quad \text{and} & S_8^i &= \frac{1}{4}(1+\xi^i)(1+\eta^i)(1+\zeta^i); \end{aligned}$$

and vector $\mathbf{e}^i$ contains the position coordinates of the eight nodes of the element i. This element's deformation is quantified using Green-Lagrange strains. The enhanced assumed strains are introduced in a similar manner to the ANCF shell element, but using a fully three-dimensional distribution for such strains. This distribution is defined by the linear matrix $\mathbf{N}$ as

$$\mathbf{N}(\xi) = \begin{bmatrix} \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \zeta & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi & \zeta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta & \zeta \end{bmatrix}. \tag{1}$$

Note that nine internal parameters, α, are used for the EAS brick element. Details on this matrix and the general application of EAS to brick elements may be found in Ref. [1]. This element can be used for simulating the following material constitutive behavior –among others:

- Linear isotropic material (large deformation) (see ANCF white paper)
- Composite material (see ANCF white paper)
- Mooney-Rivlin solid

Mooney-Rivlin material

Chrono's API allows for dynamically simulation a variety of bodies that can experience rigid and deformation motion. Of special interest to some engineering applications is the Mooney-Rivlin material, which describes a quasi-incompressible solid that can experience large nonlinear deformation based on a potential energy function. Rubber elasticity is usually described in terms of the invariants of the Cauchy-Green stretch tensor I_1, I_2, and I_3. Since rubber is considered an incompressible solid, the third invariant, I_3, does not play a role in its constitutive equations.

Rivlin established that the strain energy for a rubber material may generally be expressed as

$$W_R = \sum_{i,j=0}^{\infty} C_{ij} (I_1 - 3)^i (I_2 - 3)^j, \tag{2}$$

which, if only the first terms of the series are retained, yields the strain energy function characterizing the Mooney-Rivlin material:

$$W_{MR} = C_{10} (I_1 - 3) + C_{01} (I_2 - 3) \,. \tag{3}$$

C_{10} and C_{01} are material constants that need to be determined experimentally. For computational purposes, the invariants of the deviatoric Cauchy-Green stretch tensor $\bar{I}_1$ and $\bar{I}_2$ are used instead (see [2]); thereby eliminating the numerical instabilities associated with volumetric strains.

3. Validation of Eight-noded Brick Element with Enhanced Assumed Strain

3.1 Linear isotropic material

The linear, isotropic internal forces of the eight-noded brick element are validated in this section by using a plate made up of a variable number of brick elements, with one edge fully clamped to the ground. A gradually applied load acts on a tip of the plate of dimensions 1 m x 1 m x 0.01 m; a cosine function is used to smoothen the dynamic effect of the application of the load. The vertical load reaches -50 N along the vertical axis at time $t = 10$ s. This example may be modified and run in Chrono in the unit test utest_FEA_EASBrick_Iso.cpp. Figure 1 shows the evolution of the plate tip at which the load is applied. Our reference results is an `ANSYS Solid185` 100x100 mesh, which yields a total elastic displacement of -0.65426 m. Chrono meshes of 16x16 and 32x32 produce convergent results owing to the alleviation of locking through enhanced assumed strain. Moreover, Chrono's implementation shows excellent convergent features, which are demonstrated by

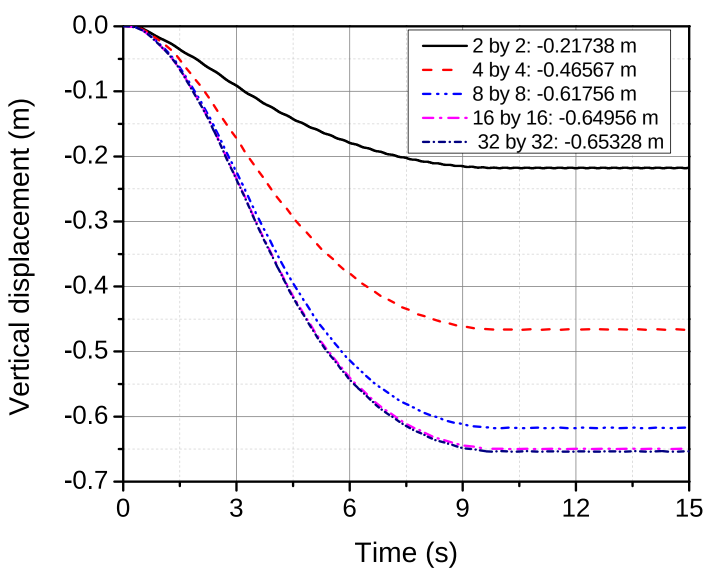

Figure 1: Dynamic evolution of a bricked plate tip under a gradually applied load of 50 N

comparison with meshes of `ANSYS`' `Solid185` (see Fig. 2).

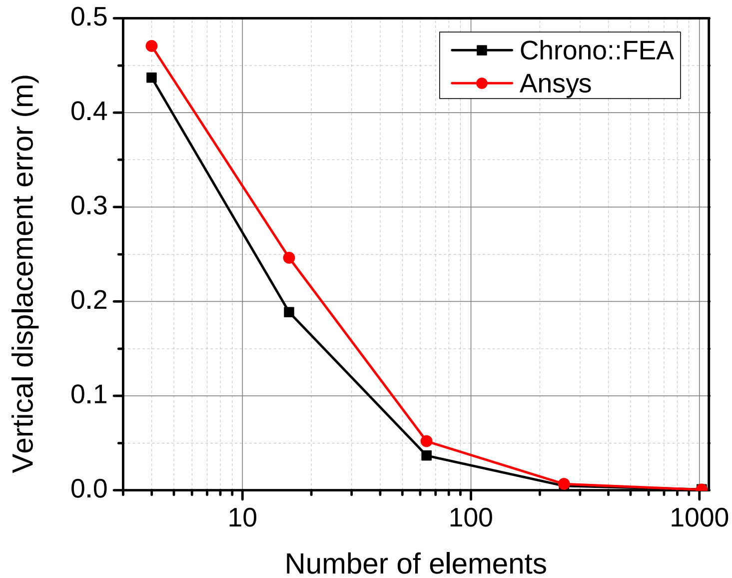

Figure 2: Convergence analysis of static deflection under the action of a load

3.2 Hyperelastic Mooney-Rivlin material

In this subsection we validate Chrono's Mooney-Rivlin material implementation outlined in Section 2. To that end, we build a model of a plate of dimensions 1 m x 1 m x 0.1 m composed of EAS brick elements. A vertical load of -50 N is applied to an upper corner of the plate's free side. This scenario may be simulated using the file utest_FEA_EASBrick_Iso.cpp, substituting the thickness of the plate

```
double plate_lenght_z = 0.1;
```

and enabling the Mooney-Rivlin material and setting the two constants $C_{10} = 50,000\,\mathrm{MPa}$ and $C_{01} = 10,000\,\mathrm{MPa}$ for each finite element:

```
element->SetMooneyRivlin(true);
element->SetMRCoefficients(50000,10000);
```

The simulations in `ANSYS` are carried out using large-deformation, static analyses; the element `Solid185` of the commercial software with enhanced assumed strain is used for comparison. The results from Chrono are obtained using structural damping and applying a ramp load, similarly to the example of the previous subsection. Good convergence may be observed in Figure 3.

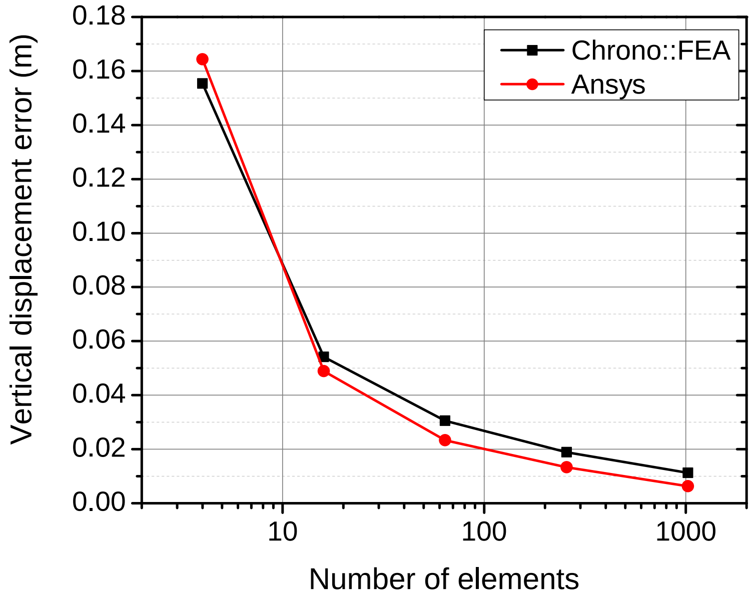

Figure 3: Convergence analysis of static deflection under the action of a load for Mooney-Rivlin material

References

[1] U. Andelfinger and E. Ramm. EAS-elements for two-dimensional, three-dimensional, plate and shell structures and their equivalende to HR-elements. *International Journal for Numerical Methods in Engineering*, 36:1311–1337, 1993.

[2] G. Orzechowski and J. Frączek. Nearly incompressible nonlinear material models in the large deformation analysis of beams using ANCF. *Nonlinear Dynamics*, 82:451–464, 2015.