Support for Multi-Physics in Chrono

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The Story Ahead

- Overview of multi-physics strategy in Chrono
  - Summary of handling rigid/flexible body dynamics using Lagrangian approach
  - Summary of handling fluid, and fluid-solid interaction using Lagrangian approach
  - Summary of handling large & nonlinear deformations using Lagrangian approach
- Strategy: fall back on Lagrangian approach to multi-physics
  - Rationale: code reuse

Chrono::Vehicle – Mobility on Granular Terrain

ANCF Shell + Initial Configuration + Internal Pressure

Question Of Interest

- Can we use similar modeling approach and same solver to handle fluid/clay/slurry?

- That is, can we use one framework to do multi-physics?

Multi-Physics Angles

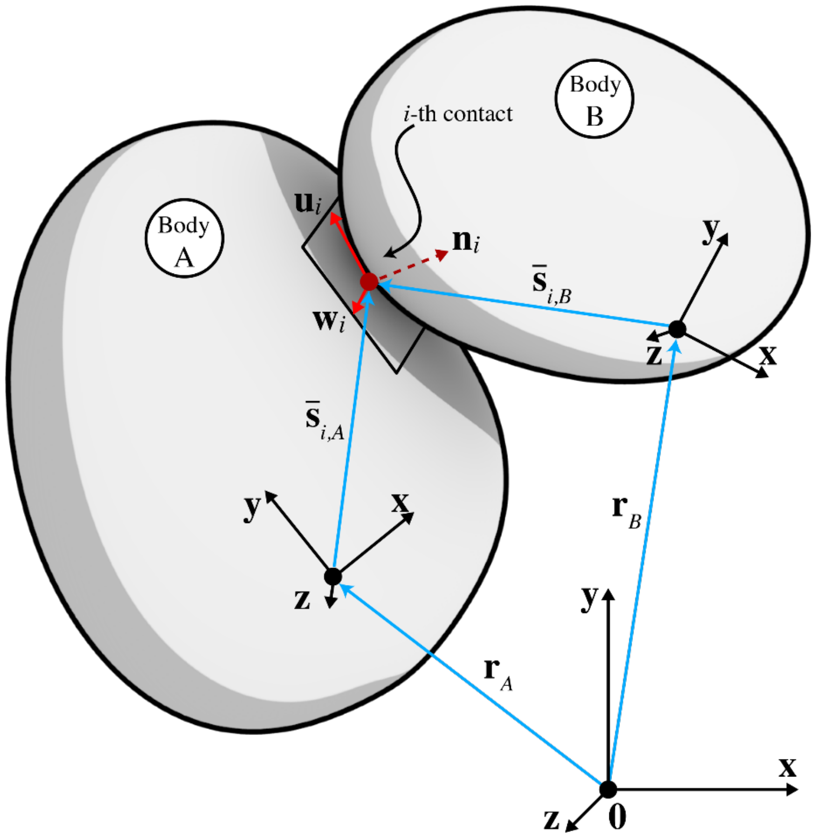
Lagrangian Viewpoint:
For both Solid and Fluid Phases

Rigid Body Dynamics

Fluid Dynamics

Deformable Body Dynamics

Discrete Element Method: Penalty & Complementarity



Computational Many-body Dynamics
Handling interactions between shapes

Complementarity
(rigid-body contact)

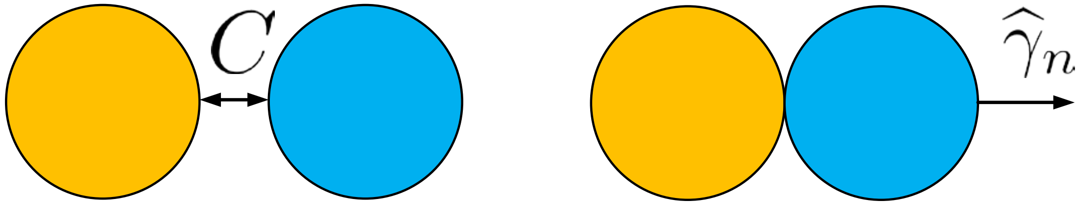
Penalty
(deformable-body contact)

Optimization techniques

Collision detection

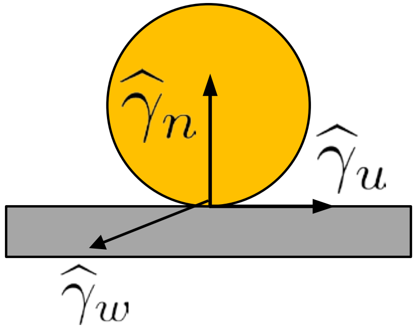
Rigid Body Dynamics

- Contact Constraints



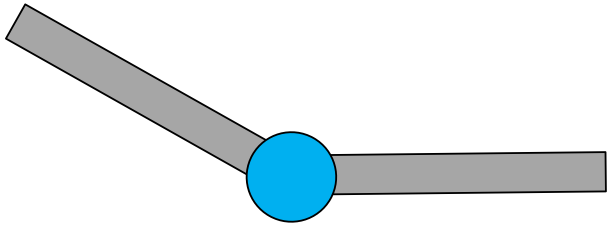
$$0 \le \widehat{\gamma}^c_{i,n} \perp \overbrace{C^c_i}^{\text{Gap function}}(\boldsymbol{x}) \ge 0$$

- Frictional Contact Constraint



$$(\widehat{\gamma}^c_{i,u}, \widehat{\gamma}^c_{i,w}) = \underset{\sqrt{(\widehat{\gamma}^c_{i,u})^2 + (\widehat{\gamma}^c_{i,w})^2} \le \mu^f_i \widehat{\gamma}^c_{i,n}}{\operatorname{argmin}} \underbrace{\boldsymbol{v}^T (\widehat{\gamma}^c_{i,u} \mathbf{D}^c_{i,u} + \widehat{\gamma}^c_{i,w} \mathbf{D}^c_{i,w})}_{\text{Friction dissipation energy}}$$

- Bilateral/Joint Constraints



$$\overbrace{\boldsymbol{C}^j(\boldsymbol{x}, t)}^{\text{Bilateral Joint Constraint}} = \mathbf{0}$$

Complementarity Approach: The Math

$$\begin{aligned}
\dot{\mathbf{q}} &= \mathbf{L}(\mathbf{q})\mathbf{v} \\
\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} &= \mathbf{f}(t, \mathbf{q}, \mathbf{v}) - \mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q}, t)\lambda + \sum_{i \in \mathcal{A}(\mathbf{q}, \delta)} \underbrace{\left(\widehat{\gamma}_{i,n}\, \mathbf{D}_{i,n} + \widehat{\gamma}_{i,u}\, \mathbf{D}_{i,u} + \widehat{\gamma}_{i,w}\, \mathbf{D}_{i,w}\right)}_{i^{th}\,\text{frictional contact force}} \\
\mathbf{0} &= \mathbf{g}(\mathbf{q}, t)
\end{aligned}$$

$$i \in \mathcal{A}(\mathbf{q}(t), \delta) \quad : \quad \begin{cases} 0 \leq \Phi_i(\mathbf{q}) \quad \perp \quad \widehat{\gamma}_{i,n} \geq 0 \\ (\widehat{\gamma}_{i,u}, \widehat{\gamma}_{i,w}) = \underset{\sqrt{(\bar{\gamma}_u^i)^2 + (\bar{\gamma}_w^i)^2} \leq \mu_i \widehat{\gamma}_{i,n}}{\operatorname{argmin}} \mathbf{v}^T \cdot \left(\bar{\gamma}_u^i\, \mathbf{D}_{i,u} + \bar{\gamma}_w^i\, \mathbf{D}_{i,w}\right) \end{cases}$$

$$\overbrace{\mathbf{q}^{(l+1)}}^{\text{Generalized positions}} = \mathbf{q}^{(l)} + \overbrace{h}^{\text{Step size}} \underbrace{\mathbf{L}}_{\text{Velocity transformation matrix}}(\mathbf{q}^{(l)})\mathbf{v}^{(l+1)}$$

$$\mathbf{M}(\overbrace{\mathbf{v}^{(l+1)}}^{\text{Generalized speeds}} - \mathbf{v}^{(l)}) = \underbrace{h\mathbf{f}(t^{(l)}, \mathbf{q}^{(l)}, \mathbf{v}^{(l)})}_{\text{Applied impulse}} - \overbrace{\mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q}^{(l)}, t)\lambda}^{\text{Reaction impulse}} + \sum_{i \in \mathcal{A}(q^{(l)}, \delta)} \underbrace{(\gamma_{i,n}\,\mathbf{D}_{i,n} + \gamma_{i,u}\,\mathbf{D}_{i,u} + \gamma_{i,w}\,\mathbf{D}_{i,w})}_{\text{Frictional contact reaction impulses}}$$

$$0 = \underbrace{\frac{1}{h}\mathbf{g}(\mathbf{q}^{(l)}, t)}_{\text{Stabilization term}} + \mathbf{g}_{\mathbf{q}}{}^{T}\mathbf{v}^{(l+1)} + \mathbf{g}_t$$

$$i \in \mathcal{A}(\mathbf{q}(t), \delta) \quad : \quad \begin{cases} 0 \leq \overbrace{\frac{1}{h}\Phi_i(\mathbf{q}^{(l)})}^{\text{Stabilization term}} + \mathbf{D}_{i,n}^{T}\mathbf{v}^{(l+1)} \perp \gamma_{i,n} \geq 0 \\ (\gamma_{i,u}, \gamma_{i,w}) = \underset{\sqrt{(\bar{\gamma}_u^i)^2 + (\bar{\gamma}_w^i)^2} \leq \mu_i \gamma_{i,n}}{\operatorname{argmin}} \mathbf{v}^{T,(l+1)} \cdot (\bar{\gamma}_u^i\,\mathbf{D}_{i,u} + \bar{\gamma}_w^i\,\mathbf{D}_{i,w}) \end{cases}$$

D.E. Stewart and J.C. Trinkle. An implicit time-stepping scheme for rigid body dynamics with inelastic collisions and coulomb friction. *IJNME*, 39:2673-2691, 1996.

$$\overbrace{\mathbf{q}^{(l+1)}}^{\text{Generalized positions}} = \mathbf{q}^{(l)} + \overbrace{h}^{\text{Step size}} \underbrace{\mathbf{L}}_{\text{Velocity transformation matrix}}(\mathbf{q}^{(l)})\mathbf{v}^{(l+1)}$$

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M. Anitescu, Optimization-based Simulation of Nonsmooth Rigid Multibody Dynamics, Math. Program. 105 (1)(2006) 113-143

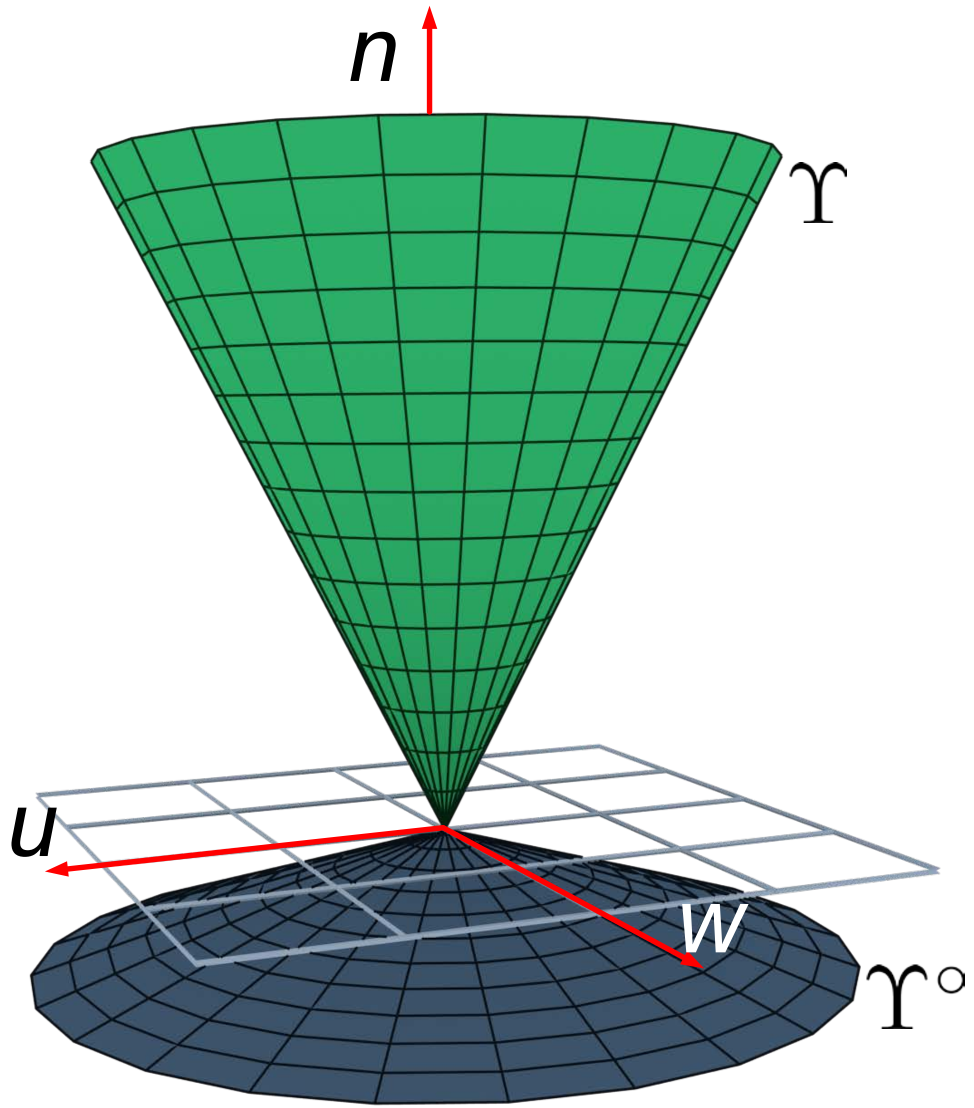
Cone Complementarity Problem (CCP)

- Contact Constraints:

$$\Upsilon_i \ni \gamma_i^c \perp -(\mathbf{N}\gamma^c + \mathbf{r})_i \in \Upsilon_i^\circ$$

- Lagrange multipliers γ_i^c should be in/on friction cone

$$\Upsilon_i = \{[x,y,z]^T \in \mathbb{R}^3 | \sqrt{y^2+z^2} \leq \mu_a^f x\}$$
$$\Upsilon_i^\circ = \{[x,y,z]^T \in \mathbb{R}^3 | x \leq -\mu_a^f \sqrt{y^2+z^2}\}$$



The Optimization Angle

- CCP represents first order optimality condition of a quadratic problem with conic constraints

$$
\begin{aligned}
\mathbf{N} &= \mathbf{D}^T\mathbf{M}^{-1}\mathbf{D} \\
\mathbf{r} &= \mathbf{b} + \mathbf{D}^T\mathbf{M}^{-1}\boldsymbol{k} \\
\gamma &\equiv \left[\gamma_1^T, \gamma_2^T, \cdots, \gamma_{N_c}^T\right]^T \in \mathbb{R}^{3N_c}
\end{aligned}
$$

$$
\gamma^\star = \underset{\substack{\gamma_i \in \Upsilon_i \\ 1 \le i \le N_c}}{\operatorname{argmin}} \left(\frac{1}{2}\gamma^T\mathbf{N}\gamma + \mathbf{r}^T\gamma \right)
$$

- $\mathbf{N} \in \mathbb{R}^{3N_c \times 3N_c}$ is symmetric and positive semi-definite
- $\mathbf{N}$ and $\mathbf{r} \in \mathbb{R}^{3N_c}$ do not depend on γ. They are computed once at beginning of each time step
- Problem has a global solution $\gamma^\star$
- Problem doesn't have a unique solution

Wrapping it Up, Complementarity Approach

- All good once the frictional contact forces at the interface between shapes are available
  - Velocity at new time step $l + 1$ computed as

$$\mathbf{v}^{(l+1)} = \mathbf{M}^{-1} \left(\mathbf{k} + \mathbf{D}\gamma \right)$$

  - Once velocity available, the new set of generalized coordinates computed as

$$\mathbf{q}^{(l+1)} = \mathbf{q}^{(l)} + h\mathbf{L}(\mathbf{q}^{(l)})\mathbf{v}^{(l+1)}$$

Fluid-Solid Interaction

The Equations of Motion, Coupled Fluid-Solid Problem

- Solid phase: Newton-Euler equations of motion, we've already seen them

- Fluid Phase: continuity and momentum balance

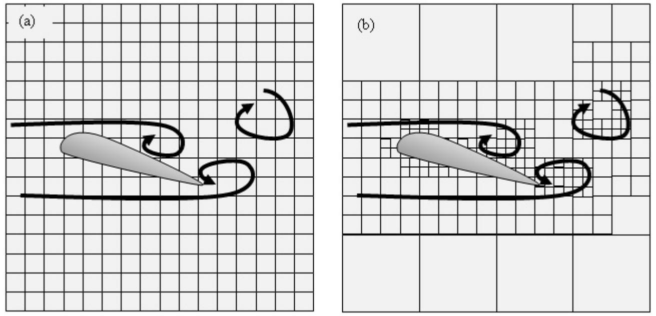
$$\frac{D\rho}{Dt} = -\rho\,\nabla\cdot\mathbf{v}$$

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p + \mu\Delta\mathbf{v} + \mathbf{b}$$

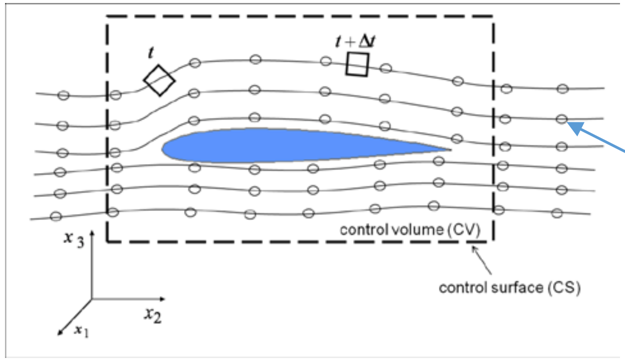
- Question: how can I cast problem above in the MBD complementarity framework?

Computational Fluid Dynamics, Two Perspectives

- Eulerian, grid-based, take on the problem
  - Approximate solution available at the grid nodes



- Lagrangian, particle-based, take on the problem
  - Approximate solution available at the location of the moving particles

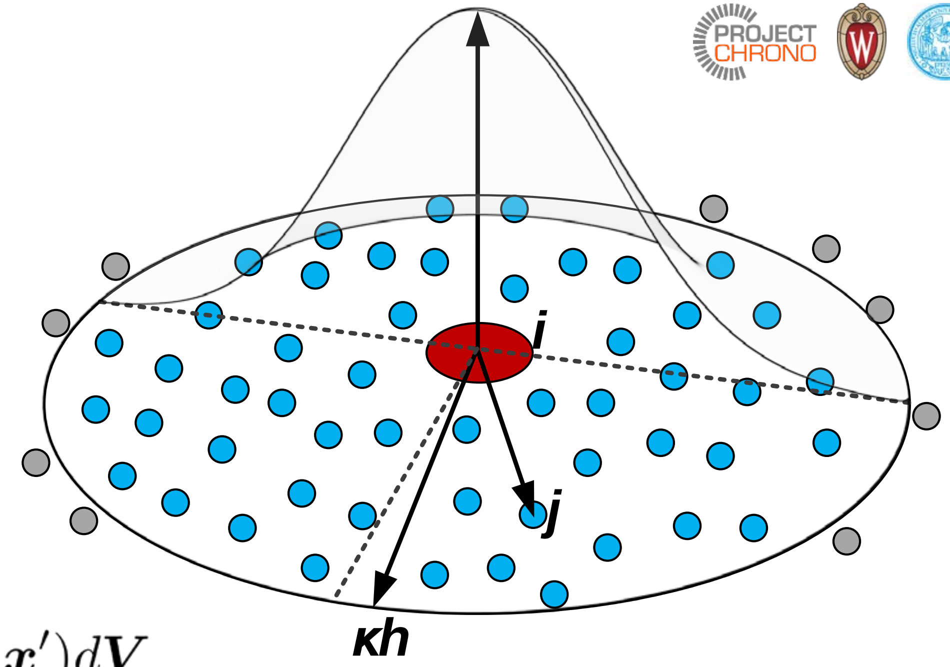


Moving particle

Credit, pics: Intel & https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/Differential_Analysis_of_Fluid_Flow#/media/File:Fluid_Dynamics_1.png

Chrono's Take on Fluid Dynamics

- Smoothed Particle Hydrodynamics
  - Quantities computed by summing up contributions from surrounding particles (weighted sum)



$$f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')\delta(\boldsymbol{x}-\boldsymbol{x}')d\boldsymbol{V}$$

$$\Downarrow$$

$$f(\boldsymbol{x}) \approx \int_{\Omega} f(\boldsymbol{x}')\, W(\boldsymbol{x}-\boldsymbol{x}', h)d\boldsymbol{V}$$

$$\Downarrow$$

$$f(\boldsymbol{x}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j)\, W(\boldsymbol{x}-\boldsymbol{x}_j, h) \quad \text{and} \quad \nabla f(\boldsymbol{x}) \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\boldsymbol{x}_j)\, \nabla W(\boldsymbol{x}-\boldsymbol{x}_j, h)$$

Function value evaluation

Function gradient evaluation

Navier-Stokes w/ SPH method

Continuity: $$\frac{d\rho_a}{dt} = \rho_a \sum_b \frac{m_b}{\rho_b} (\mathbf{v}_a - \mathbf{v}_b) \cdot \nabla_a W_{ab}$$

Momentum (Navier-Stokes): $$\frac{d\mathbf{v}_a}{dt} = -\sum_b m_b \left[\left(\frac{p_a}{{\rho_a}^2} + \frac{p_b}{{\rho_b}^2} \right) \nabla_a W_{ab} - \frac{(\mu_a + \mu_b)\mathbf{x}_{ab} \cdot \nabla_a W_{ab}}{\bar{\rho}_{ab}^2 (x_{ab}^2 + \varepsilon \bar{h}_{ab}^2)} \mathbf{v}_{ab} \right] + \mathbf{f}_{FSI} + \mathbf{f}_a$$

Lagrangian Kinematics: $$\frac{d\mathbf{x}_a}{dt} = \mathbf{v}_a$$

Weakly Compressible: $$p = \frac{{c_s}^2 \rho_0}{\gamma} \left\{ \left(\frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right\}$$

XSPH: $$\hat{\mathbf{v}}_a = \mathbf{v}_a + \Delta \mathbf{v}_a, \; \Delta \mathbf{v}_a = \zeta \sum_b \frac{m_b}{\bar{\rho}_{ab}} (\mathbf{v}_b - \mathbf{v}_a) W_{ab}$$

Shepard Filtering: $$\rho_a = \sum_b m_b W_{ab}$$

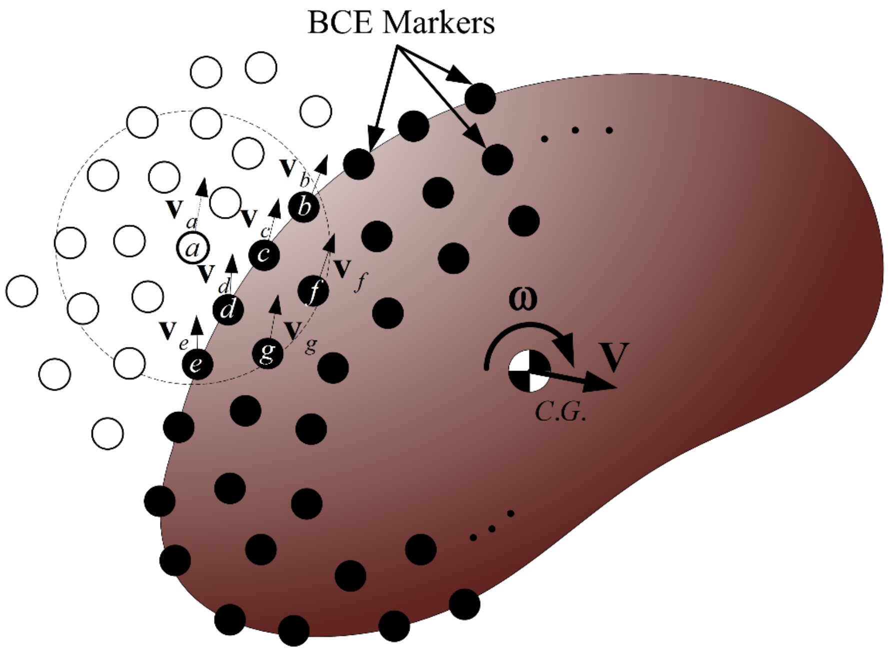
- J. Monaghan, Smoothed particle hydrodynamics, Reports on Progress in Physics 68 (1) (2005) 1703-1759.
- M. Liu, G. Liu, Smoothed particle hydrodynamics (SPH): An overview and recent developments, Archives of Computational Methods in Engineering 17 (1) (2010) 25-76.

Fluid-Solid Coupling

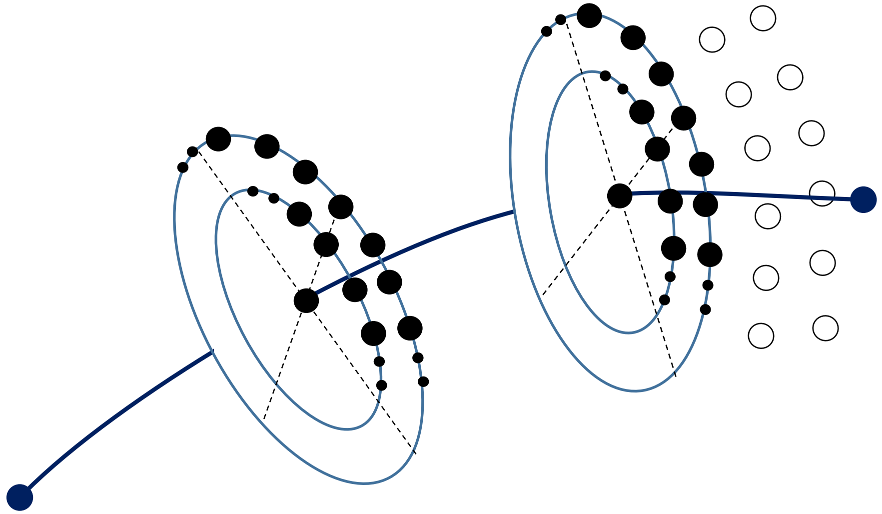
Boundary Condition Enforcing (BCE) markers for no-slip condition

- Rigidly attached to the solid body (hence their velocities are those of the corresponding material points on the solid)
- Hydrodynamic properties from the fluid

Rigid bodies/walls



Flexible beams



Interacting Rigid and Flexible Objects in Channel Flow

Fluid:

$\rho = 1000\ \mathrm{kg/m^3}$

$\mu = 1\ \mathrm{N\ s/m^2}$

$(l_x, l_y, l_z) = (1.4, 1, 1)\ \mathrm{m}$

$Re = 45$

Ellipsoids:

$\rho_s = 1000\ \mathrm{kg/m^3}$

$(a_1, a_2, a_3) = (2.25, 2.25, 3)\ \mathrm{cm}$

$N_r = 2000$

$Re_p = 2$

Beams:

$\rho_s = 1000\ \mathrm{kg/m^3}$

$E = 0.2\ \mathrm{MPa}$

$a = 1.5\ \mathrm{cm}$

$l = 64\ \mathrm{cm}$

$N_f = 40$

$n_e = 4$

Chrono::FSI

Fluid:

$\rho = 1000\ \mathrm{kg/m^3}$
$\mu = 1\ \mathrm{N\ s/m^2}$
$(l_x, l_y, l_z) = (1.4, 1, 1)\ \mathrm{m}$
$Re = 45$

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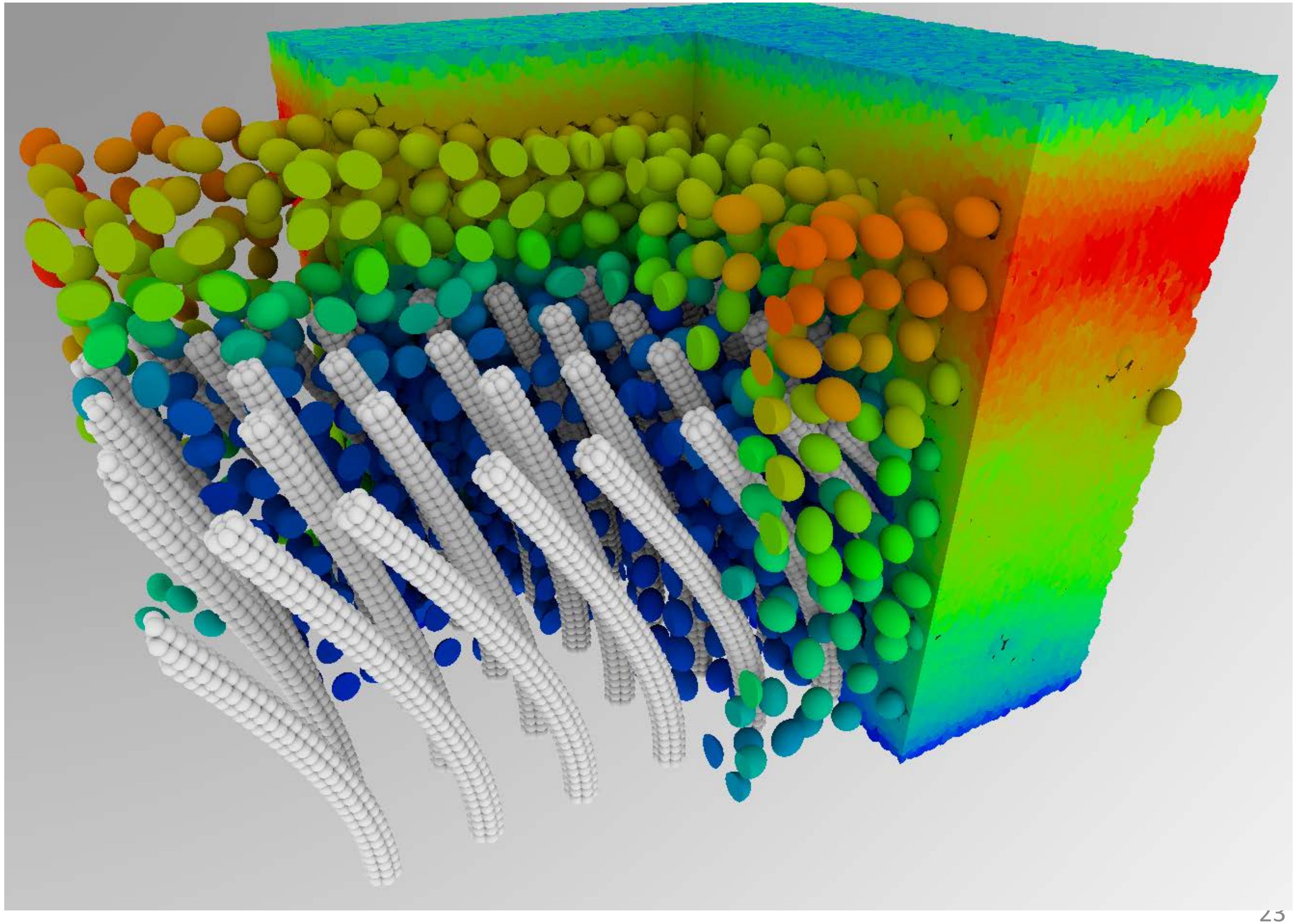
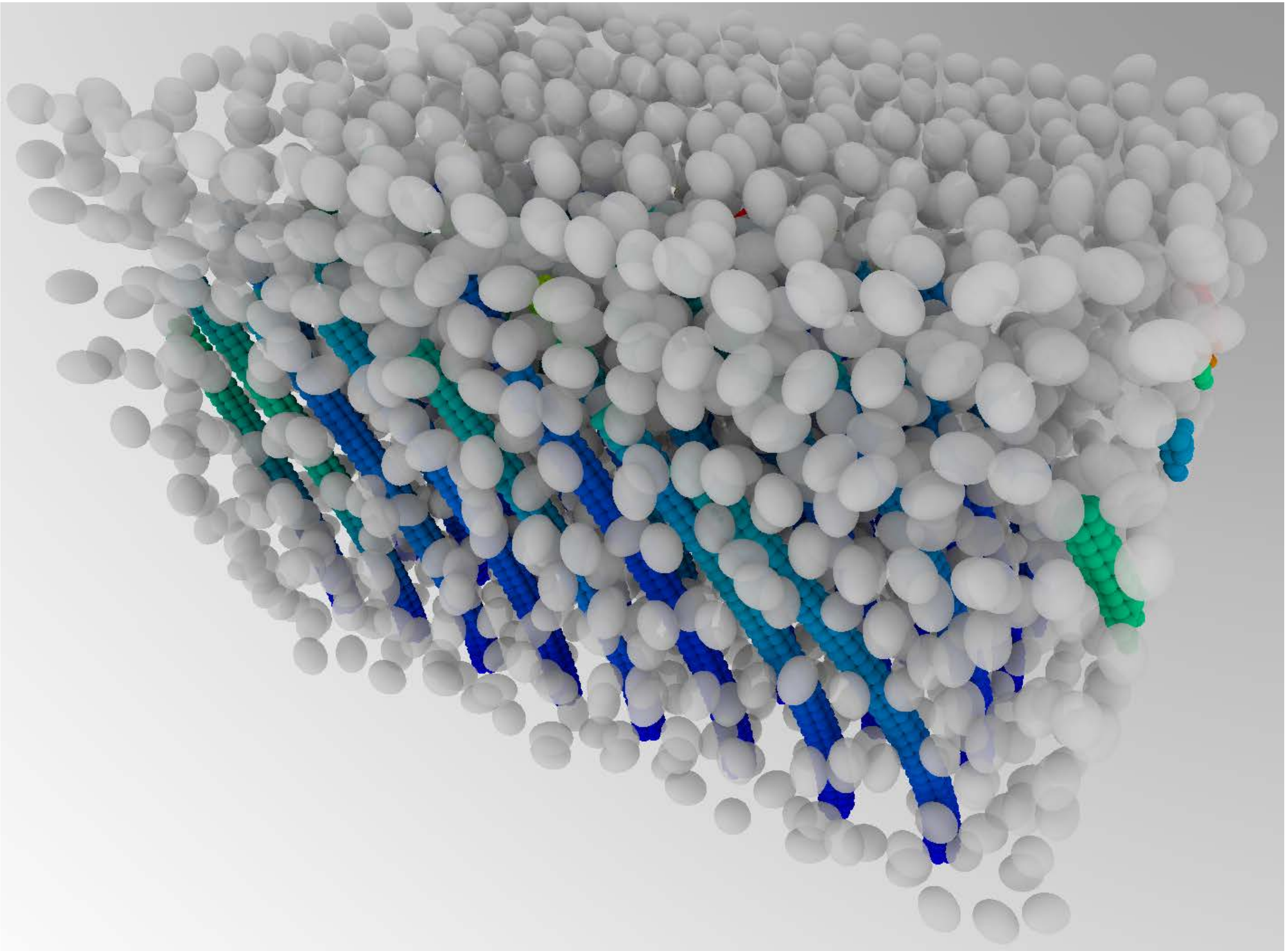
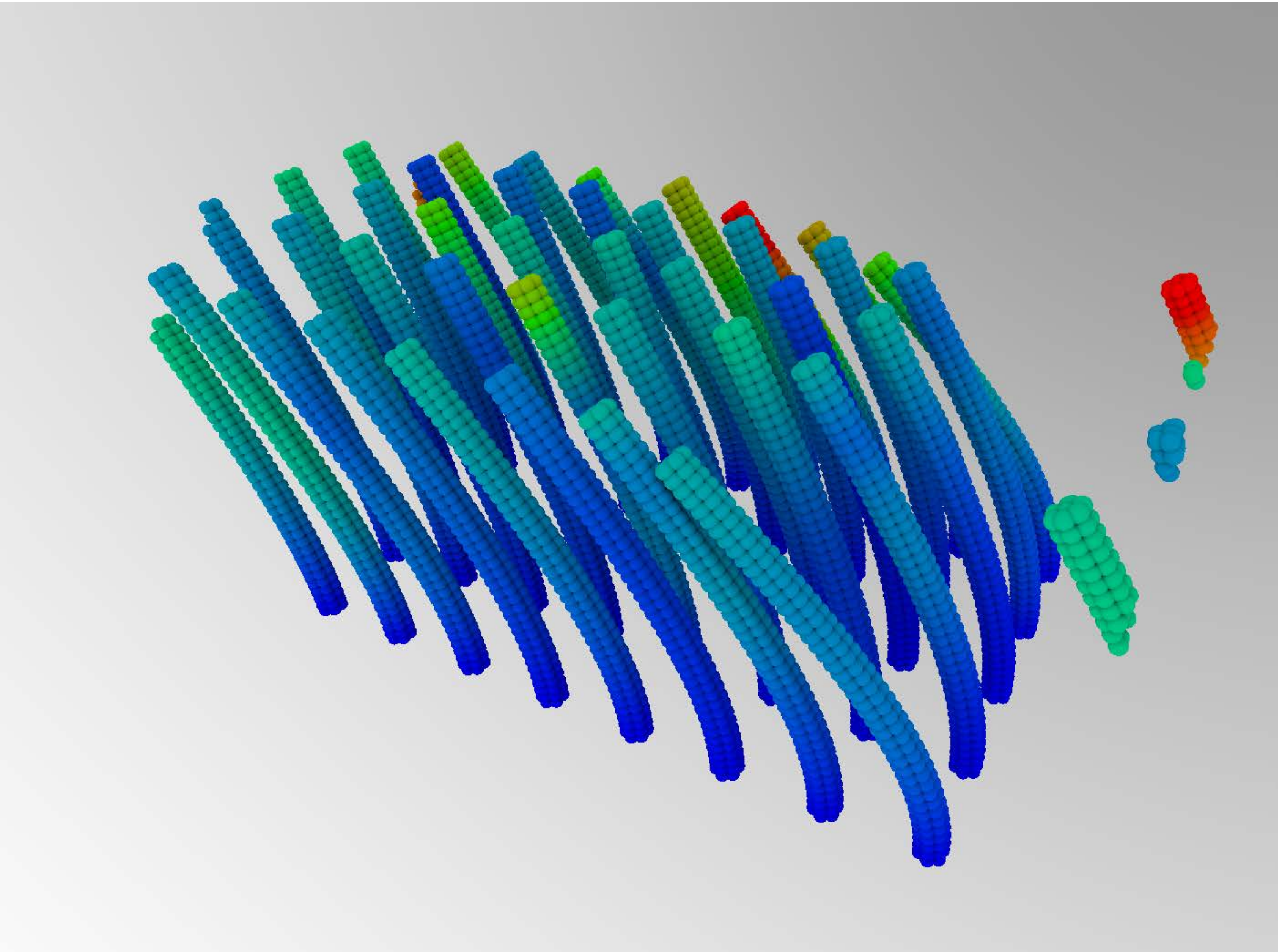






“A numerical study of the effect of particle properties on the radial distribution of suspension in pipe flow,” A. Pazouki and D. Negrut, Computers & Fluids , 108, 1-12, 2015

Pros/Cons, SPH CFD w/ Explicit Integration

- Pros
  - Straight forward to implement, simply recycle code from granular dynamics
  - Fluid-solid interaction: relatively easy to support
  - Handles free surface flows well

- Cons
  - Only weakly enforcing incompressibility of the fluid
  - Integration steps-size is very small since equation of state induces stiffness into the problem
    - Exactly like how penalty induces stiffness in frictional-contact
  - Medium accuracy, an attribute associated w/ SPH in general

Alternative Approach: Constrained Fluids

- Basic idea: use holonomic kinematic constraints to enforce incompressibility

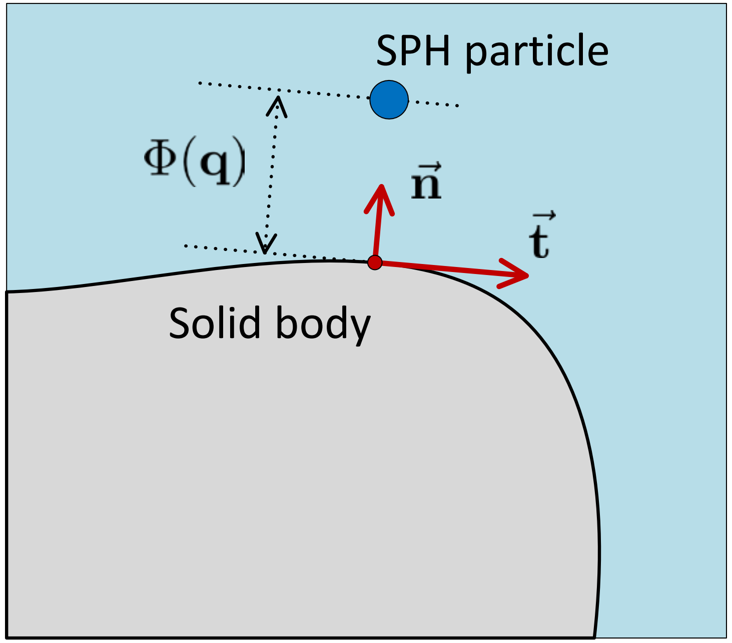
$$\cancel{p = \frac{{c_s}^2 \rho_0}{\gamma}\left\{\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right\}} \qquad C_i^f = \frac{\rho_i - \rho_0}{\rho_0} = \frac{\rho_i}{\rho_0} - 1$$

- Couples easily with impulse-velocity based formulation
  - Surface friction, cohesion, and compliant contacts possible

K. Bodin, C. Lacoursiere, M. Servin, 2012, "Constraint Fluids", *IEEE Transactions on Visualization & Computer Graphics*

Fluid-Solid Interaction, Tangent Plane

- Nonpenetration boundary conditions
  - An SPH particle cannot penetrate the rigid wall



- For contact event k, non-penetration condition modeled by inequality constraint involving signed gap function $\Phi_k(\mathbf{q})$ and normal impulse $\hat{\gamma}_{k,n} = 0$ experienced by SPH particle

$$0 \leq \Phi_k(\mathbf{q}) \quad \perp \quad \widehat{\gamma}_{k,n} \geq 0$$

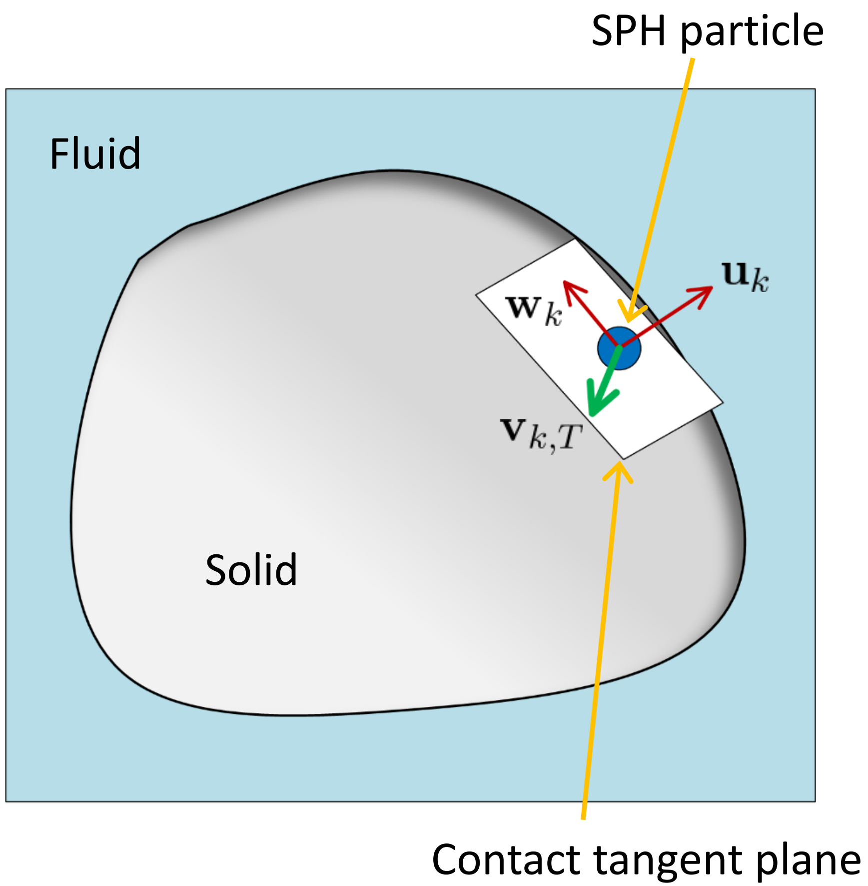
Fluid-Solid Interaction, Tangent Plane

- Interaction in the tangent plane between SPH particle and solid:

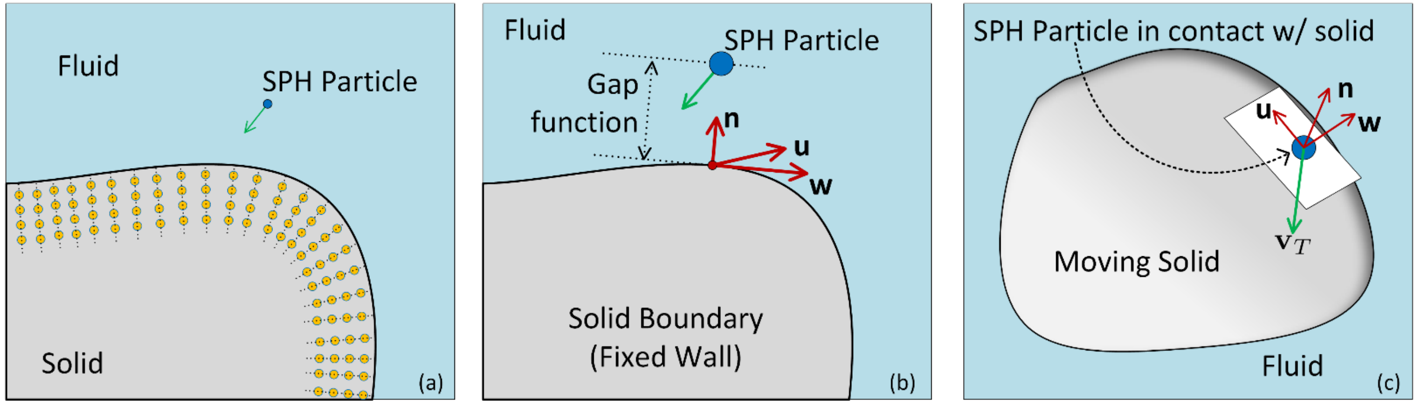
$$\mathbf{F}_{k,T}^{T} \cdot \mathbf{v}_{k,T} = -\|\mathbf{F}_{k,T}\| \, \|\mathbf{v}_{k,T}\|$$

$$0 \leq \|\mathbf{v}_{k,T}\| \perp \left(\mu_i \widehat{\gamma}_{k,n} - \sqrt{\widehat{\gamma}_{k,u}^2 + \widehat{\gamma}_{k,w}^2}\right) \geq 0$$

- Notation, for contact event k:
  - $\mathbf{v}_{k,T} \rightarrow$ relative tangential velocity at point of contact
  - $\mathbf{F}_{k,n} \rightarrow$ normal contact force
  - $\mathbf{F}_{k,T} = \mathbf{u}\widehat{\gamma}_{k,u} + \mathbf{w}\widehat{\gamma}_{k,w} \rightarrow$ friction force in the tangent plane spanned by $\mathbf{u}_k$ and $\mathbf{w}_k$
  - $\mu_k \rightarrow$ friction coefficient



Enforcing Coupling for SPH in Chrono



Fluid Dynamics Equations of Motion

$$\mathbf{x}^{(l+1)} = \mathbf{x}^{(l)} + \Delta t \mathbf{v}^{(l+1)} \tag{1}$$

Fluid-Solid Interaction, Punch Line

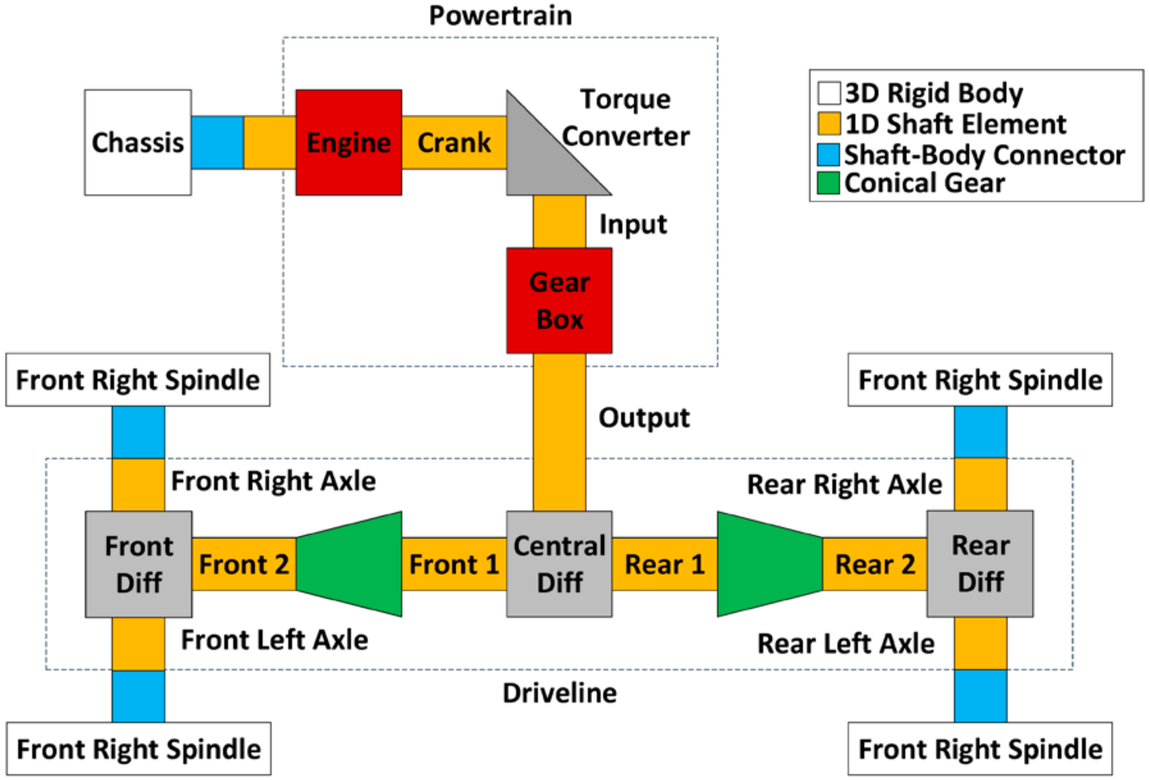
- Mathematically speaking, "constrained-fluid" approach and granular dynamics lead to same problem:

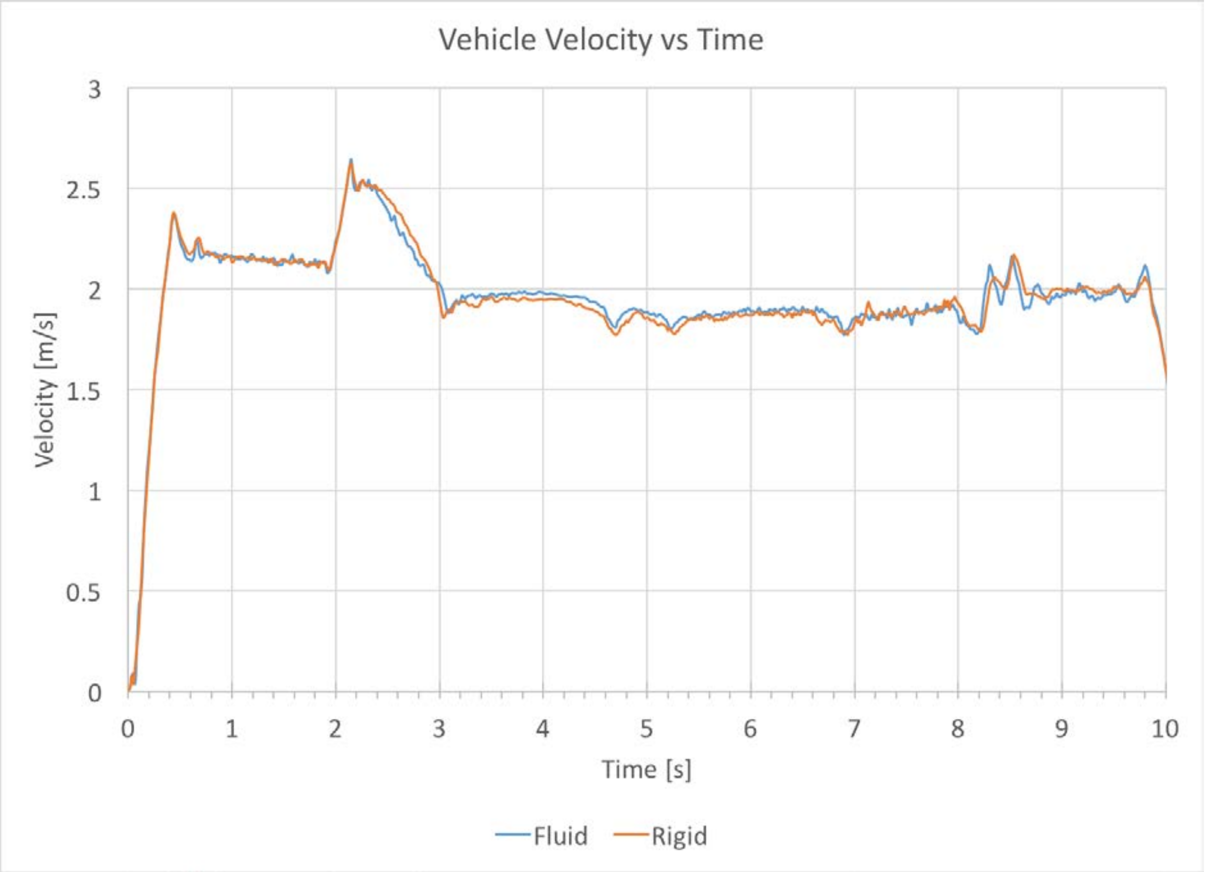
$$\gamma^\star = \underset{\substack{\gamma_i \in \Upsilon_i \\ 1 \le i \le N_F}}{\operatorname{argmin}} \left(\frac{1}{2} \gamma_F^T \, \mathbf{N}_F \, \gamma_F + \mathbf{p}_F^T \, \gamma_F \right)$$

  - Quadratic optimization with conic constraints
  - Subscript "F" to indicate this is for "fluid"
  - $\mathbf{N}_F \in \mathbb{R}^{3N_F \times 3N_F}$ is symmetric and positive semi-definite
  - $\mathbf{N}_F$ and $\mathbf{p}_F \in \mathbb{R}^{3N_F}$ do not depend on γ_F. They are computed once at beginning of each time step
  - Problem has a global solution $\gamma_F^\star$
  - Problem doesn't have a unique solution

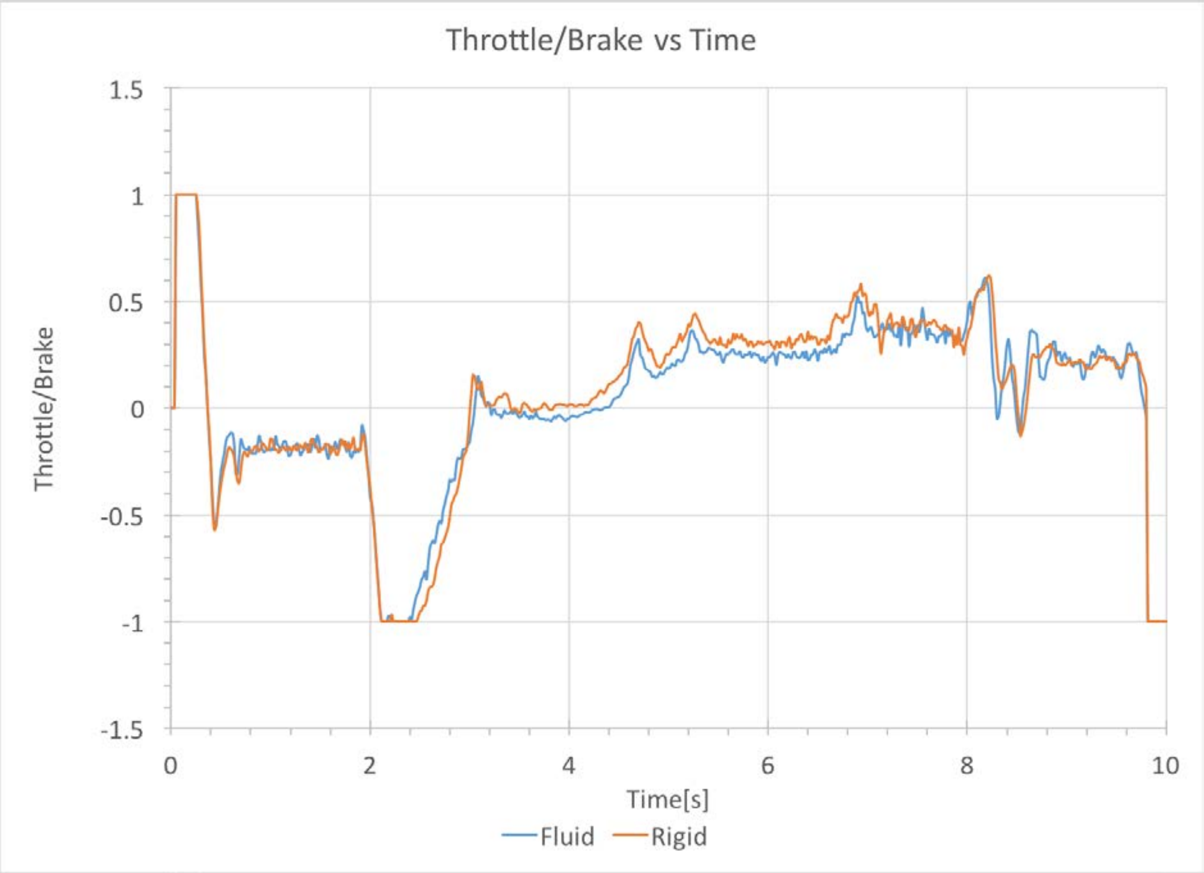
Fording Simulation

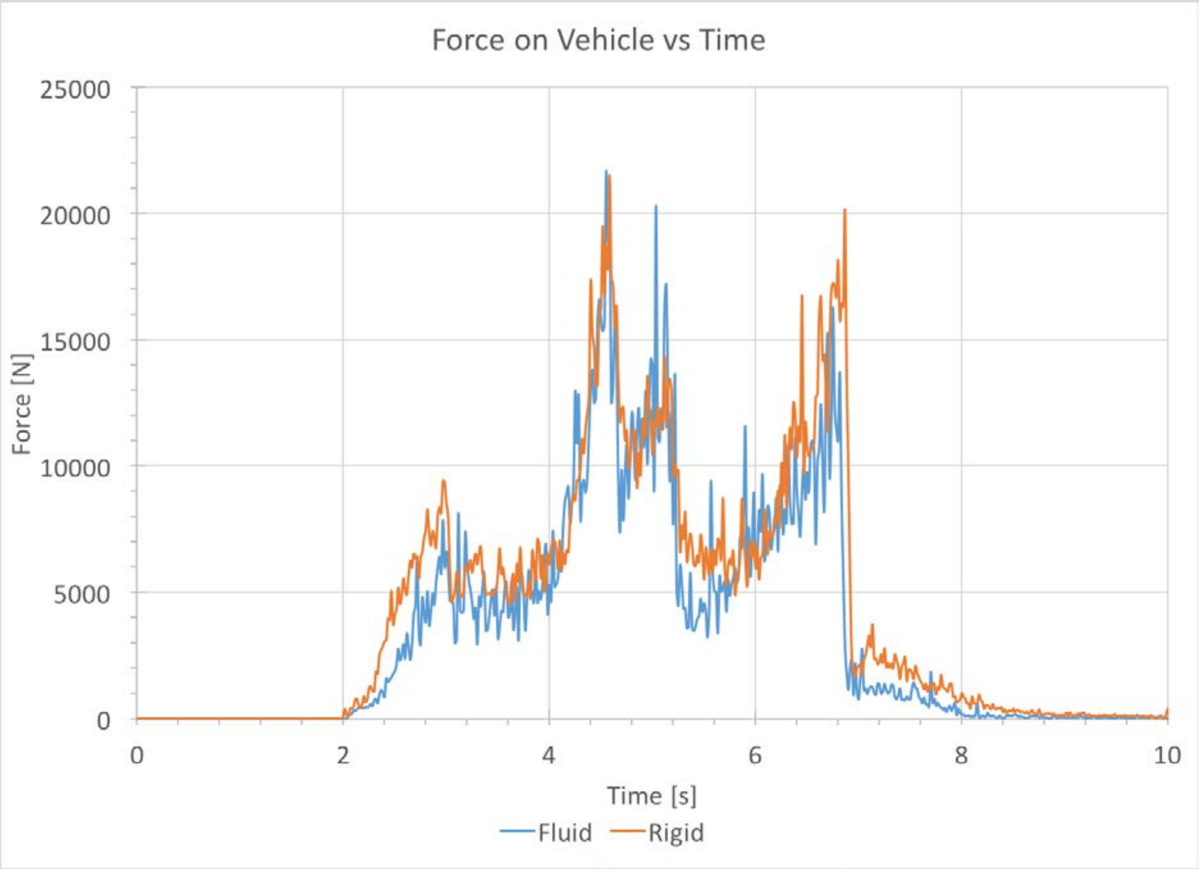
Fording, HMMWV





Fording, HMMWV

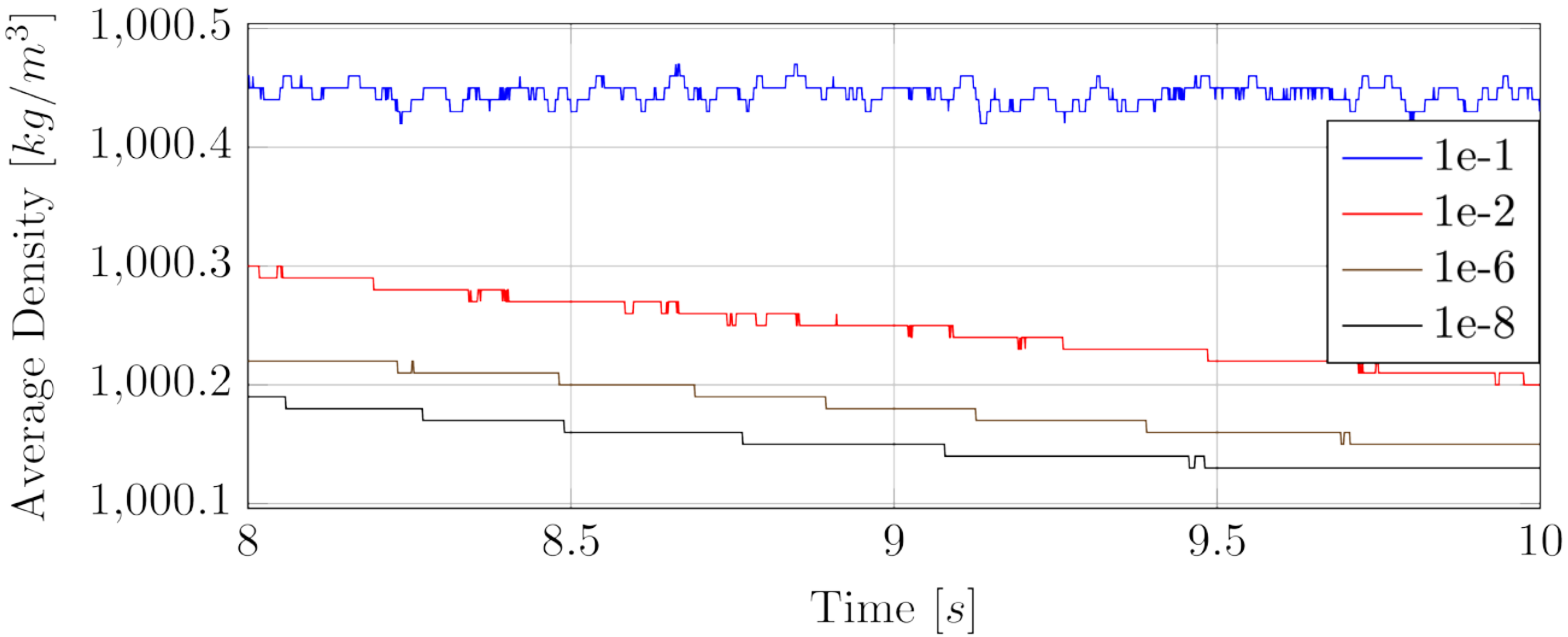




Sloshing

Incompressibility Test

Tolerance	Average Iterations	Final Density $[\frac{kg}{m^3}]$	Density Error [%]	Final Kinetic Energy $[J]$
1e-1	43	1000.43	0.043	0.85
1e-2	114	1000.2	0.02	0.82
1e-6	531	1000.15	0.015	0.59
1e-8	639	1000.13	0.013	0.44



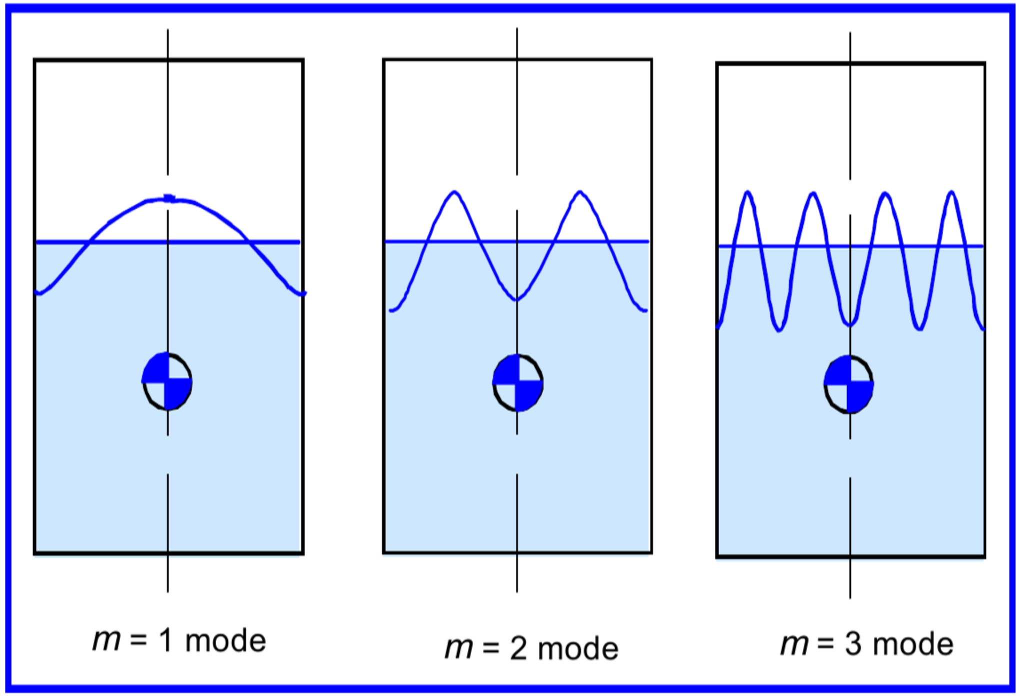
Sloshing Validation

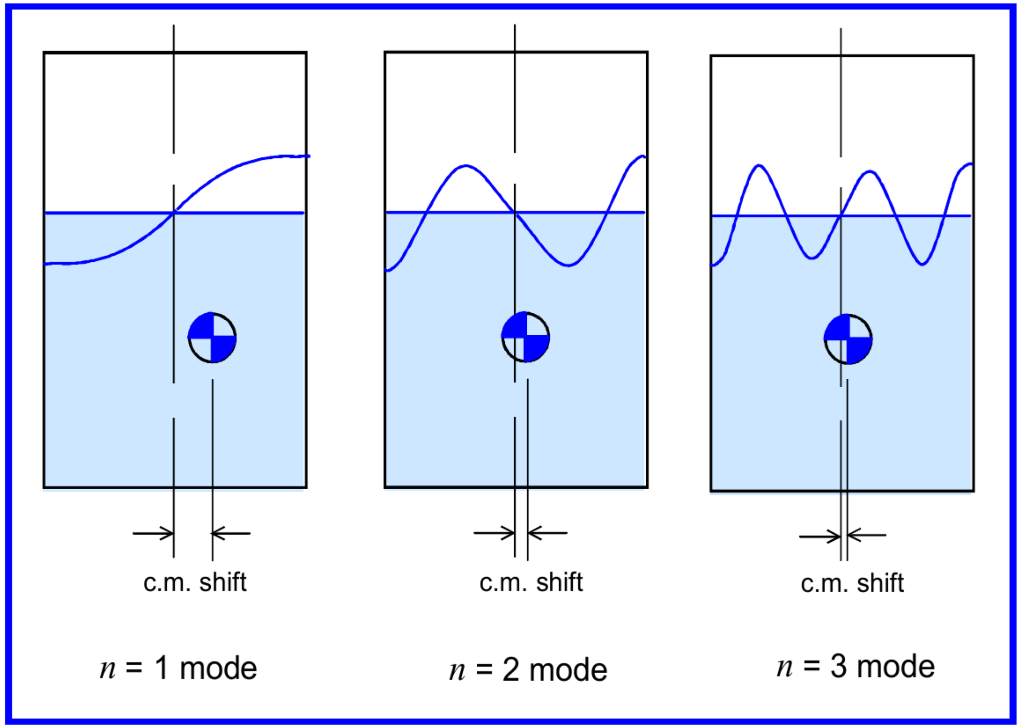
- Analytical solution to slosh dynamics

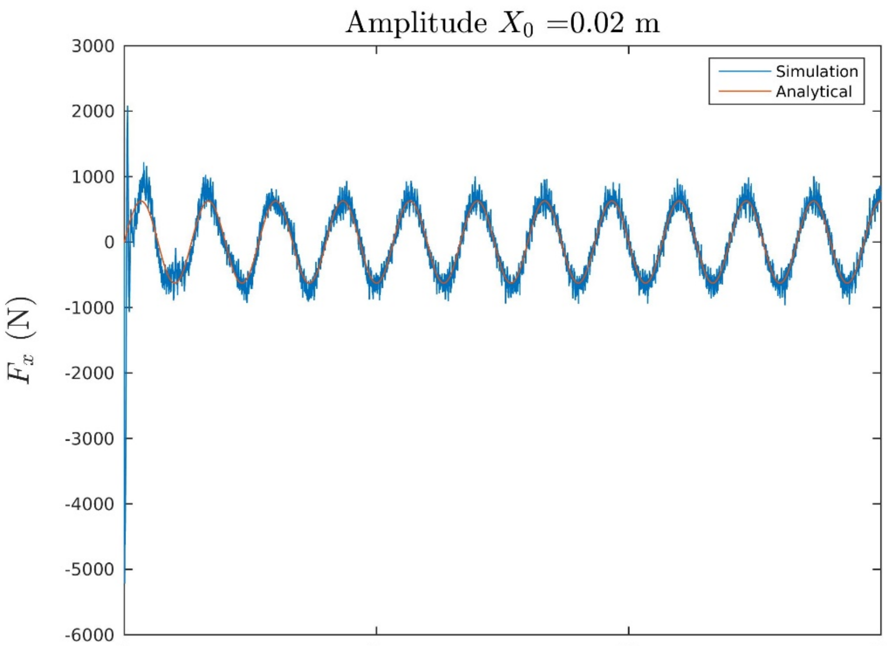
$$\omega_n^2 = (2n-1)\pi\left(\frac{g}{a}\right)\tanh\left[(2n-1)\pi\left(\frac{h}{a}\right)\right]$$

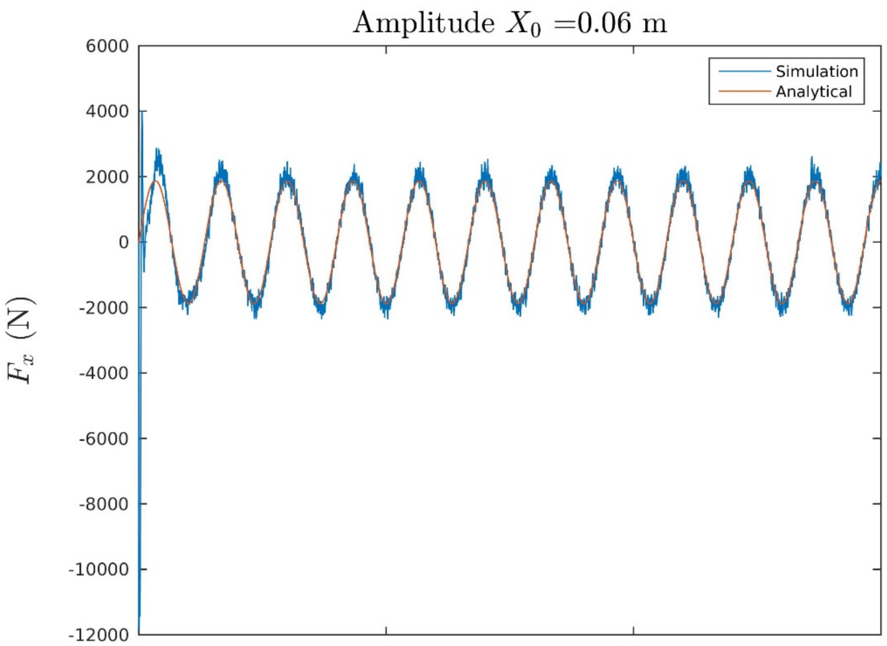
$$\omega_m^2 = 2m\pi\left(\frac{g}{a}\right)\tanh\left[2m\pi\left(\frac{h}{a}\right)\right]$$

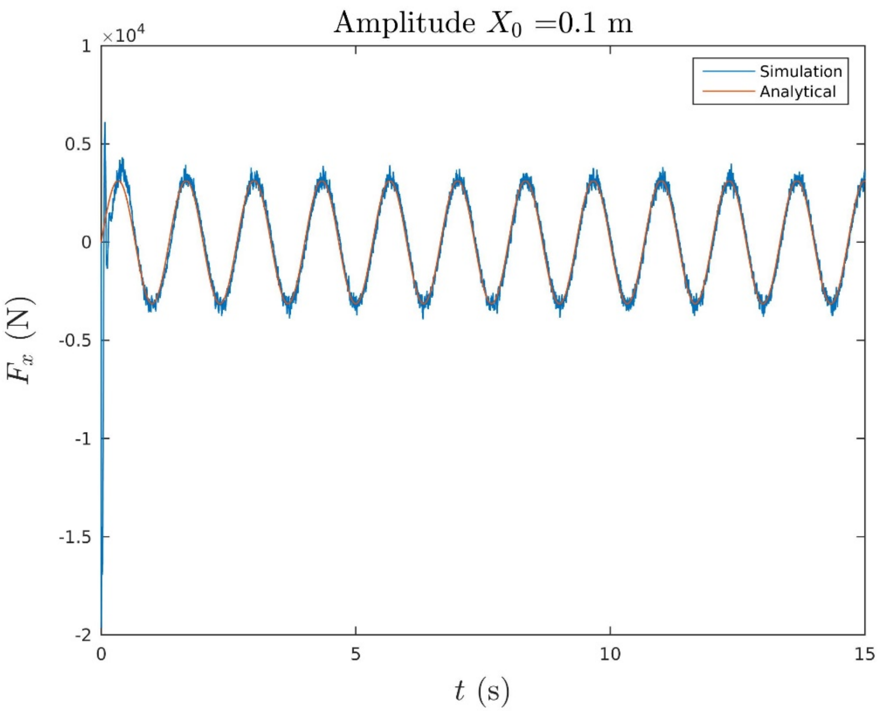
$$\frac{F_{x_0}}{\Omega^2 X_0 m_l} = 1 + 8\frac{a}{h}\sum_{n=1}^{N}\frac{\tanh\left[(2n-1)\pi h/a\right]}{(2n-1)^3\pi^3}\frac{\Omega^2}{\omega_n^2-\Omega^2}$$

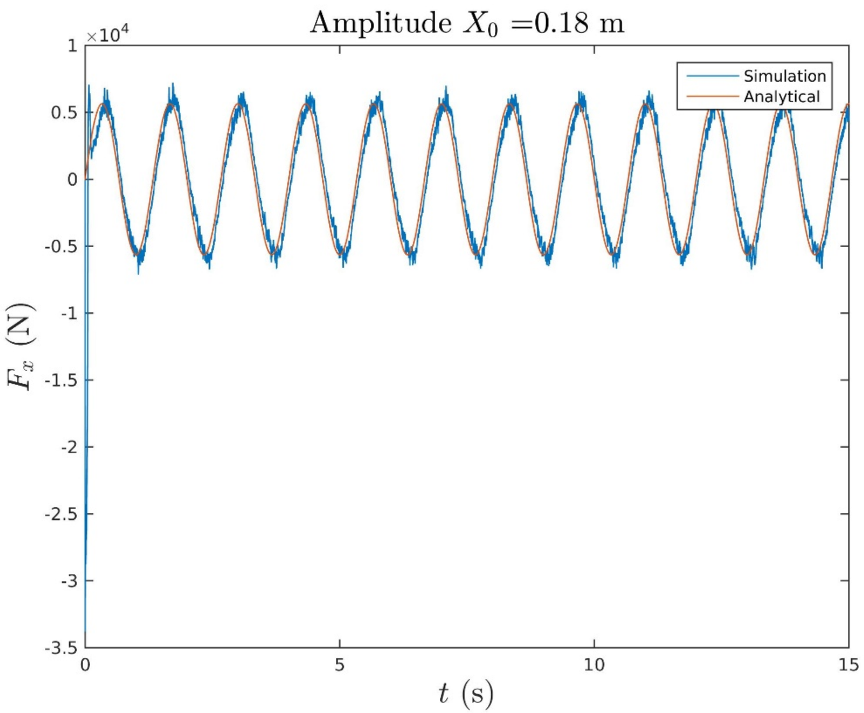


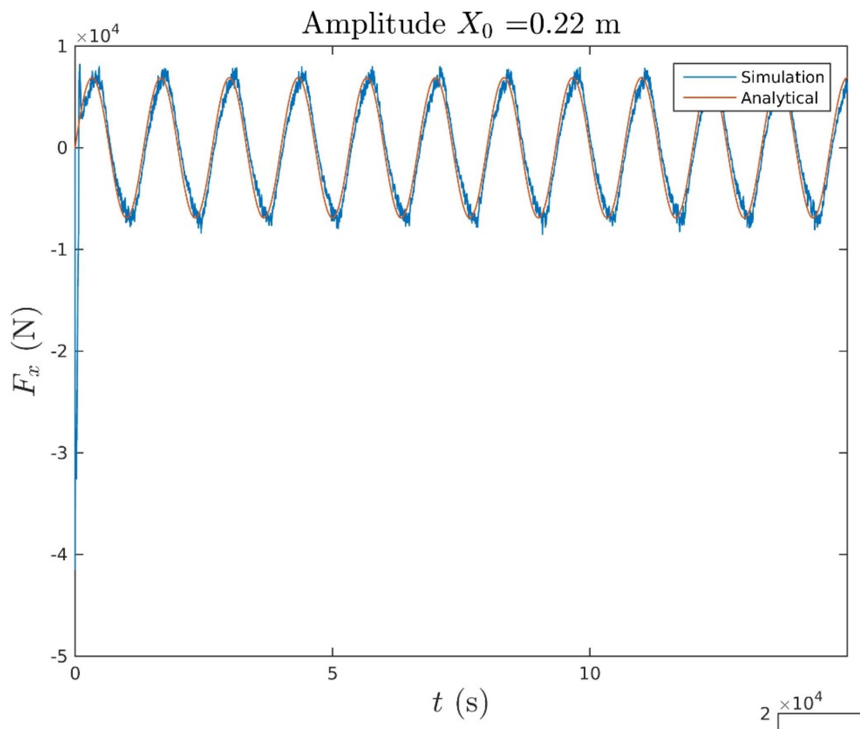


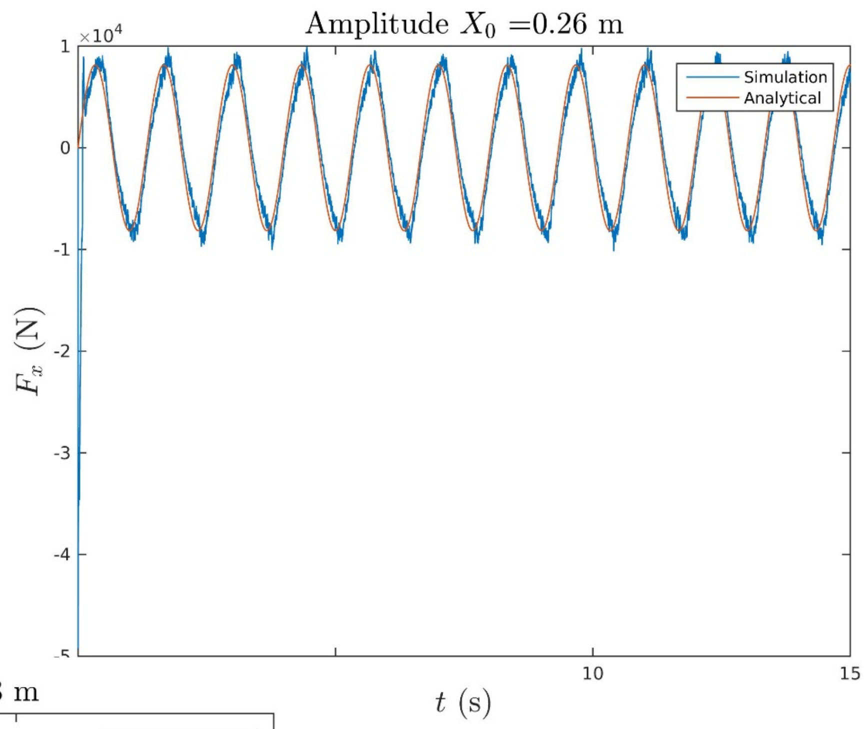

Amplitude X_0 =0.02 m
Simulation
Analytical
F_x (N)
3000
2000
1000
0
-1000
-2000
-3000
-4000
-5000
-6000

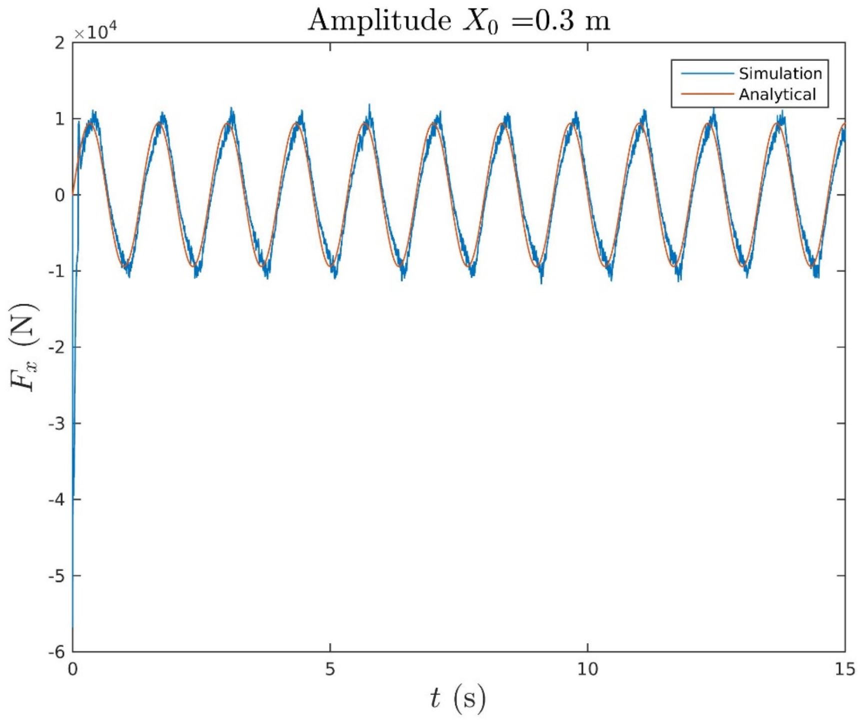

Amplitude X_0 =0.06 m
Simulation
Analytical
F_x (N)
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
-12000

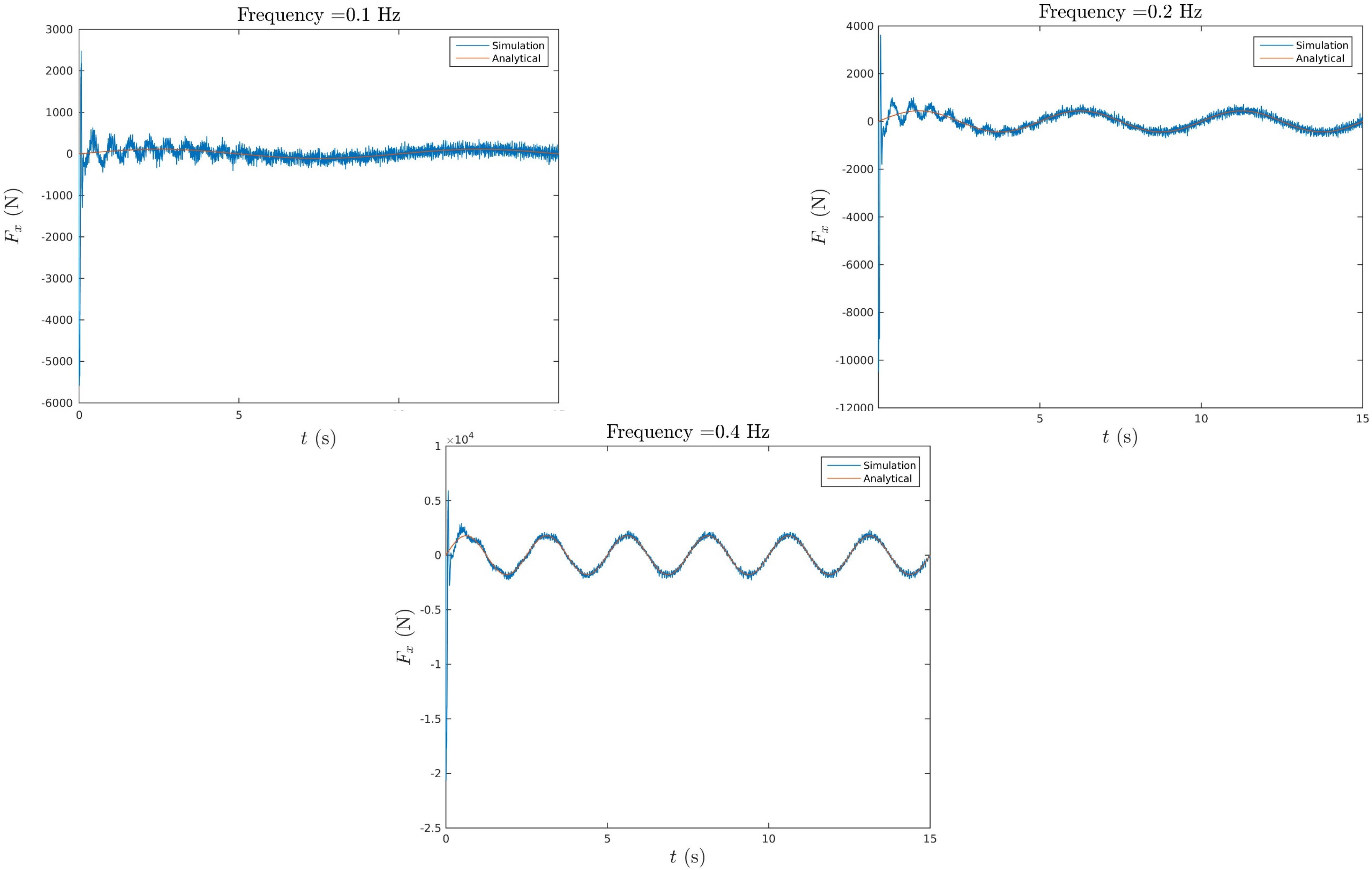

Amplitude X_0 =0.1 m
$\times 10^4$
Simulation
Analytical
F_x (N)
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
t (s)

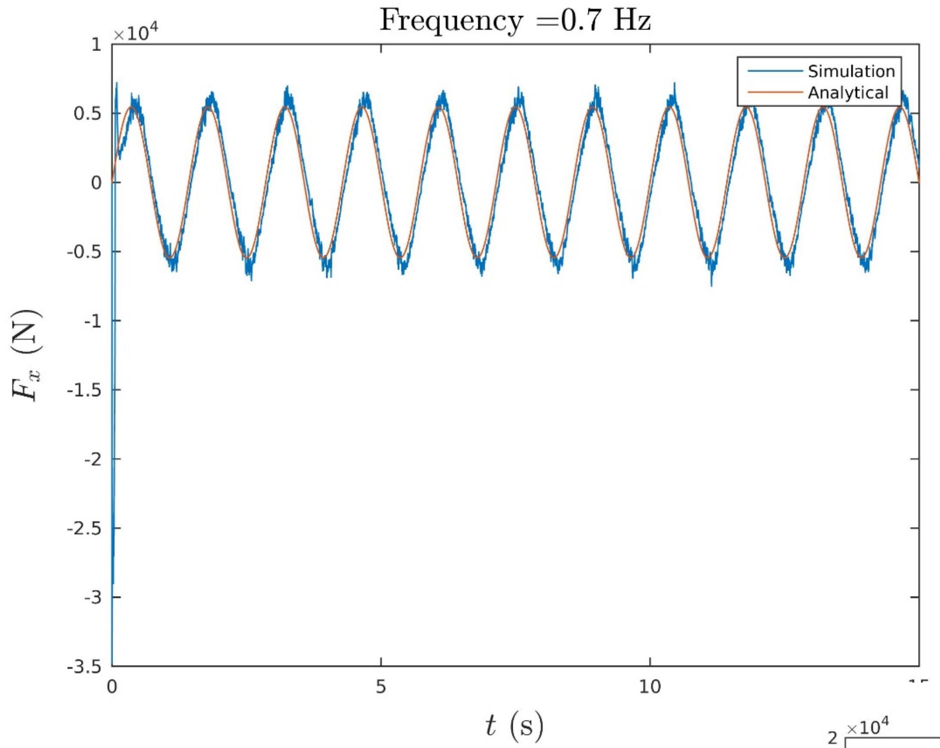

Amplitude X_0 =0.18 m
$\times 10^4$
Simulation
Analytical
F_x (N)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
5
10
15
t (s)

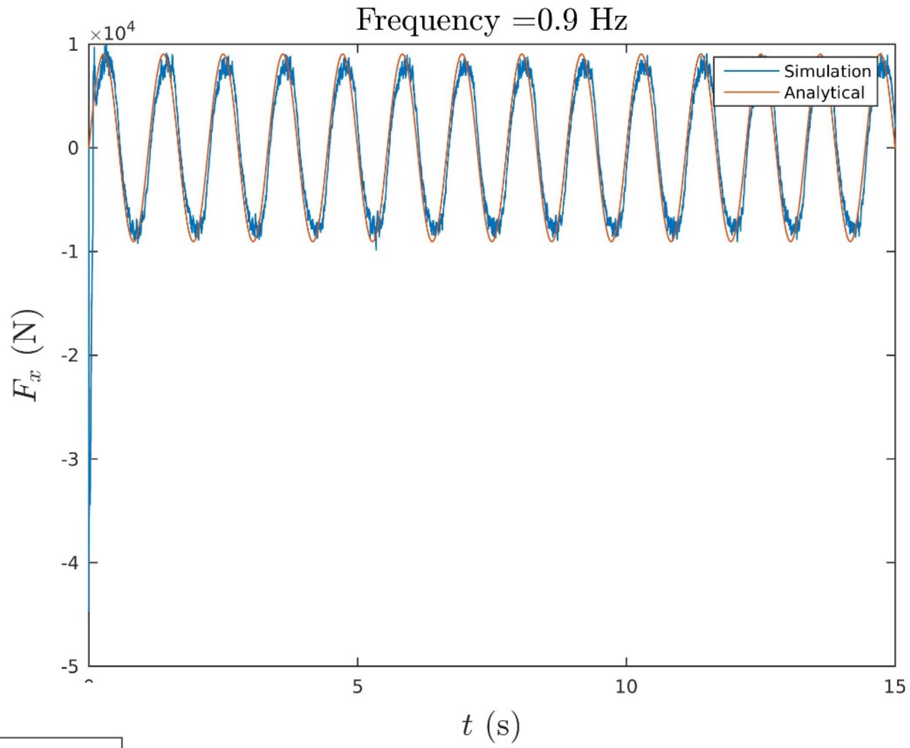

Amplitude X_0 =0.22 m
$\times 10^4$
Simulation
Analytical
F_x (N)
t (s)
1
0
-1
-2
-3
-4
-5
0
5
10

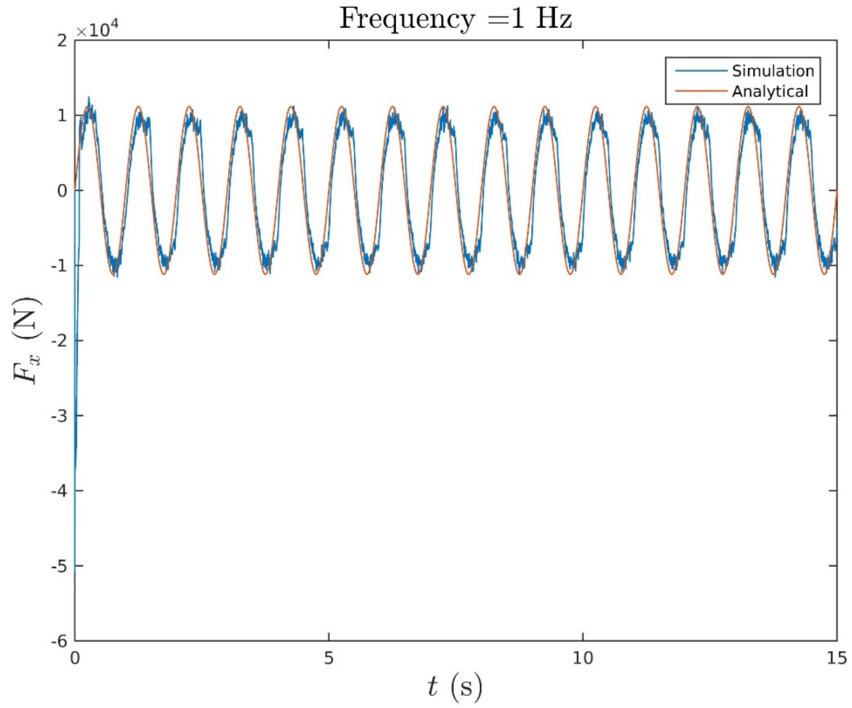

Amplitude X_0 =0.26 m
$\times 10^4$
Simulation
Analytical
F_x (N)
t (s)
1
0
-1
-2
-3
-4
10
15


Amplitude X_0 =0.3 m
$\times 10^4$
Simulation
Analytical
F_x (N)
t (s)
2
1
0
-1
-2
-3
-4
-5
-6
0
5
10
15


Frequency =0.1 Hz
Simulation
Analytical
F_x (N)
t (s)
Frequency =0.2 Hz
Simulation
Analytical
F_x (N)
t (s)
Frequency =0.4 Hz
Simulation
Analytical
$\times 10^4$
F_x (N)
t (s)


Frequency =0.7 Hz
Simulation
Analytical
F_x (N)
t (s)

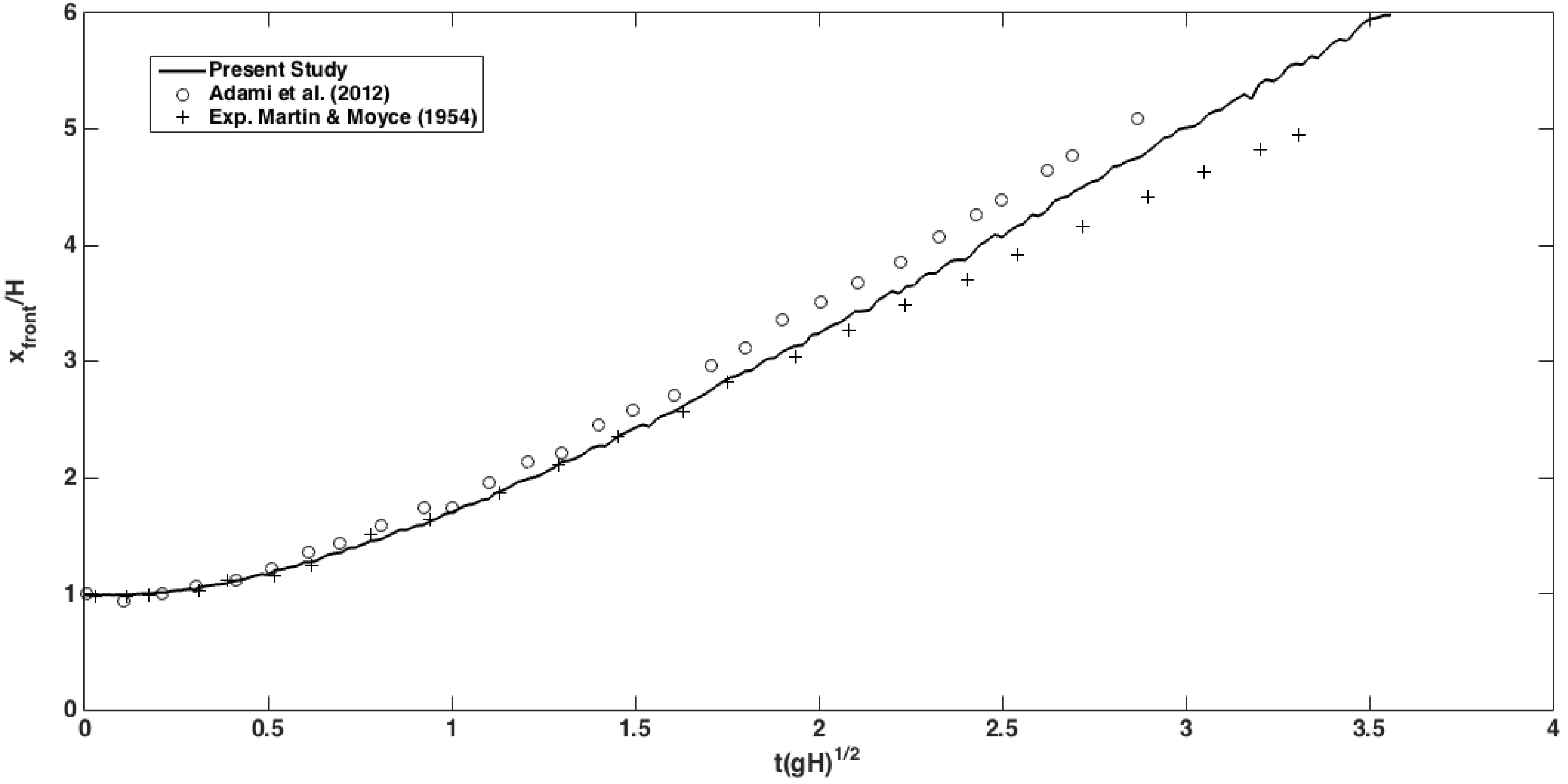

Frequency =0.9 Hz
Simulation
Analytical
F_x (N)
t (s)


Frequency =1 Hz
Simulation
Analytical
F_x (N)
t (s)

Multi-physics, robustness, speed

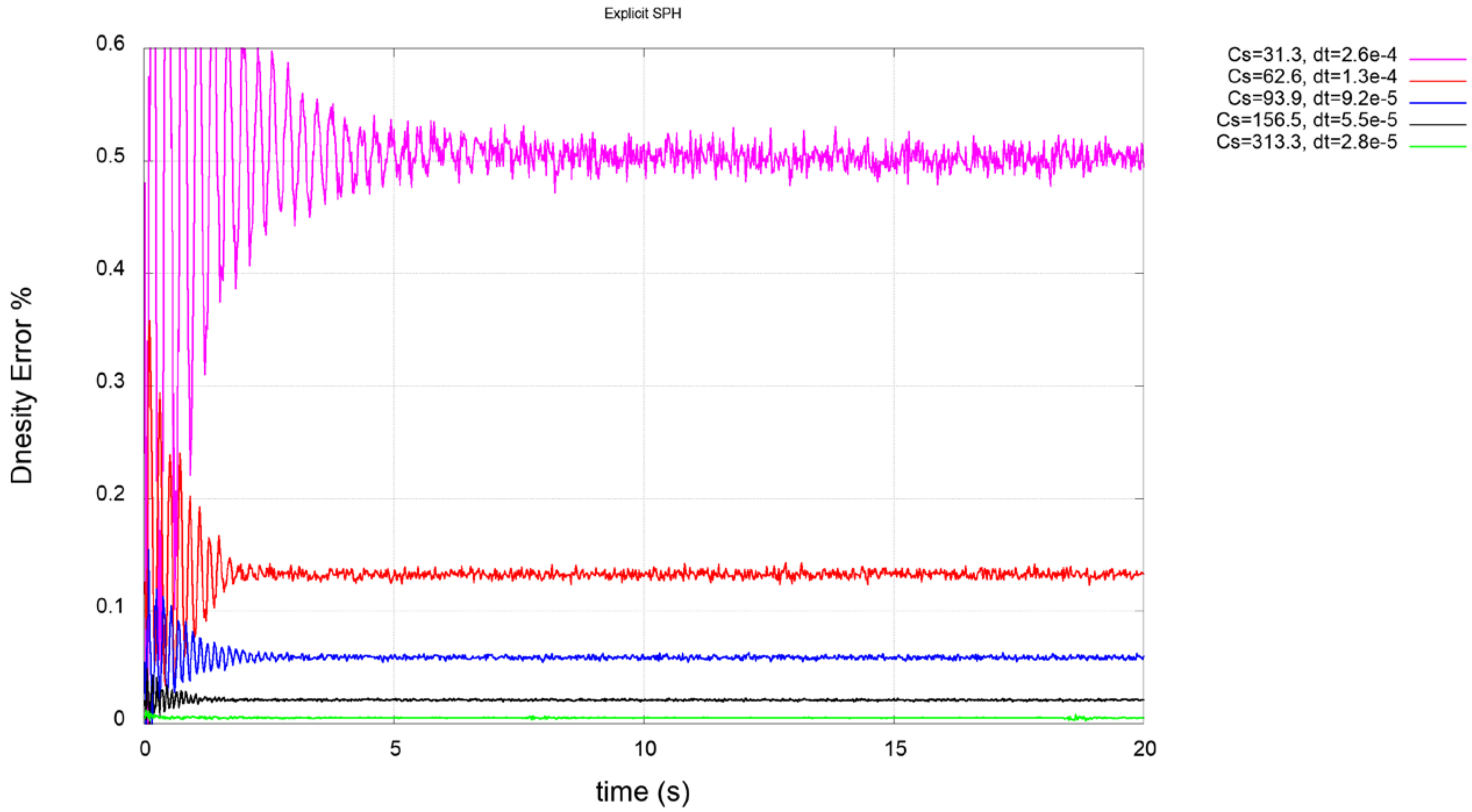
Fluid-Solid Interaction

- Constraint Fluid
- Validation of dam break simulatiom with Experimental and Explicit SPH



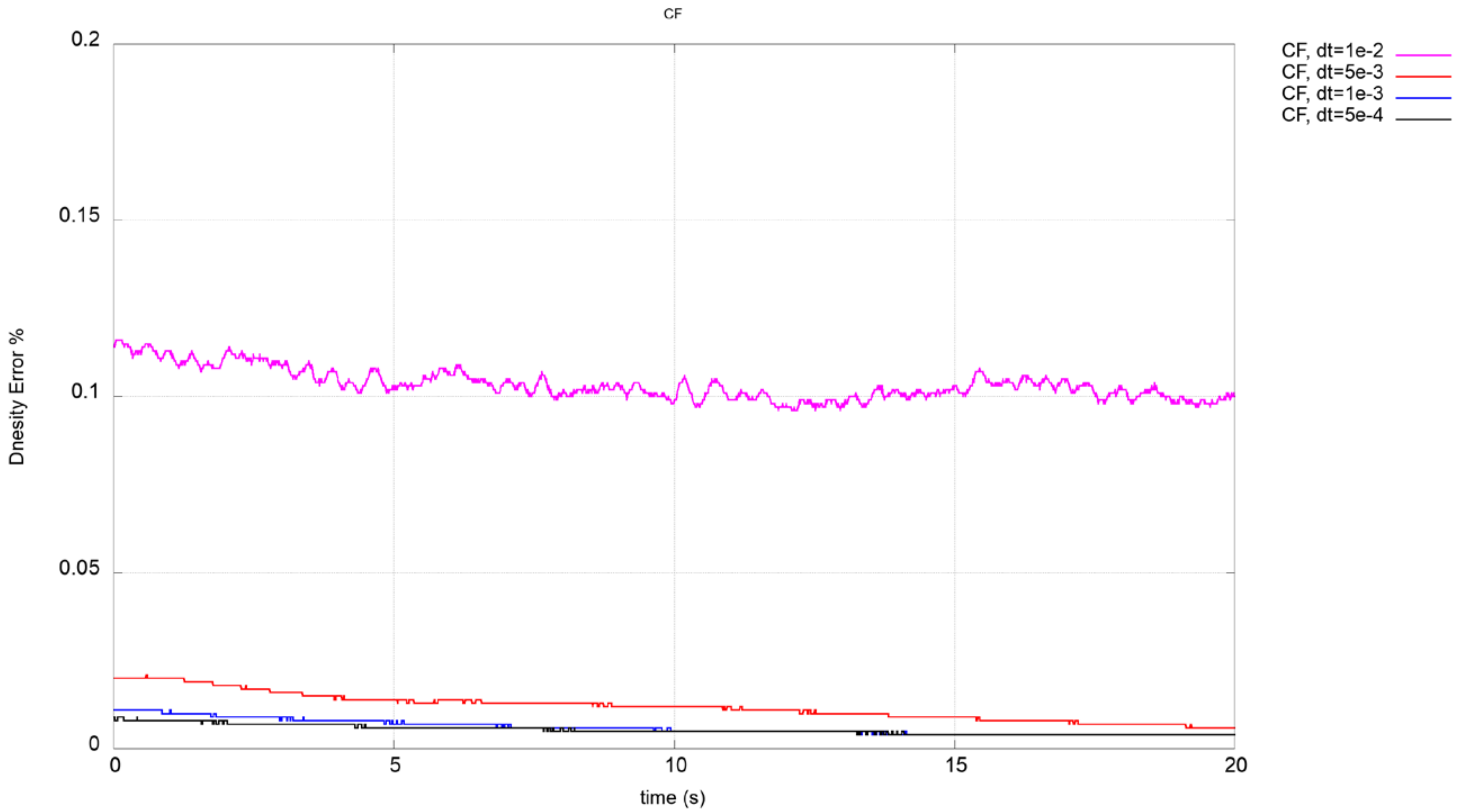
Fluid-Solid Interaction

- Compressibility Analysis (DualSphysics 4.0)



Fluid-Solid Interaction

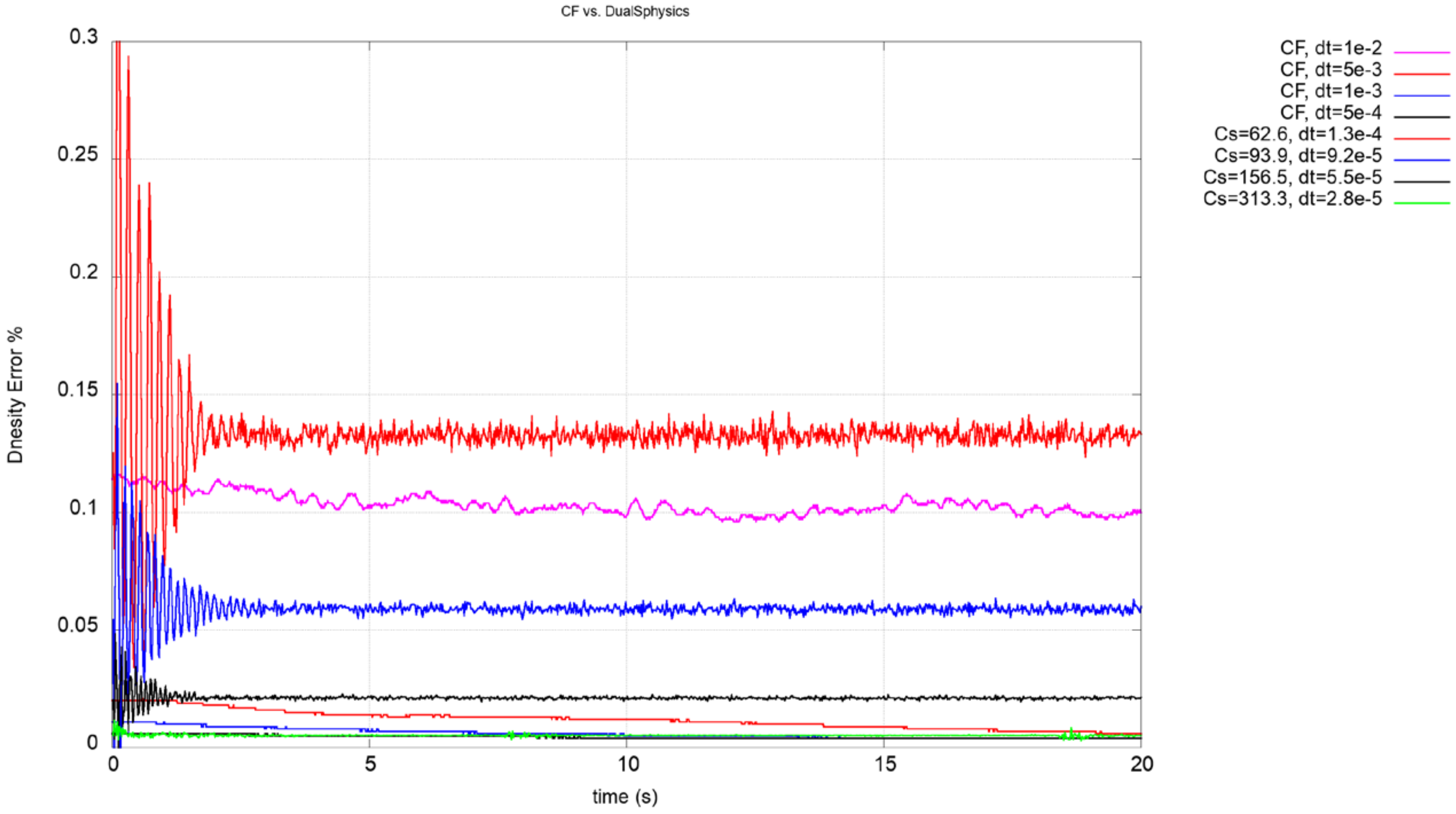
- Compressibility Analysis (Constraint Fluid)



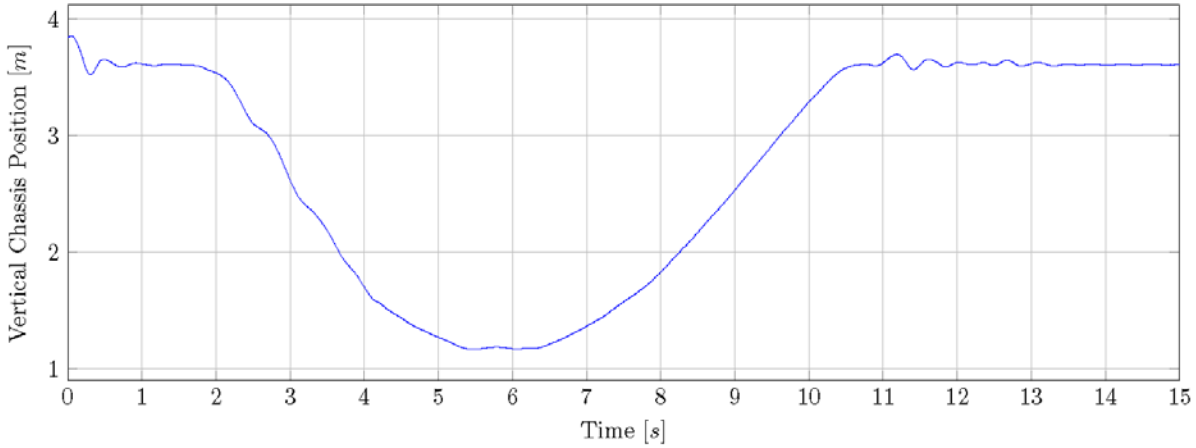
Fluid-Solid Interaction

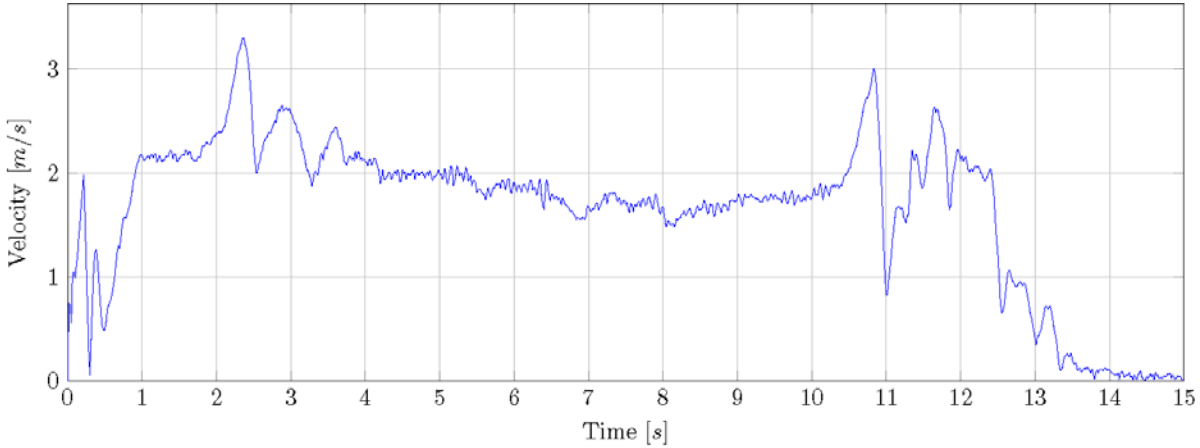
- Compressibility Analysis (Constraint Fluid + DualSphysics 4.0)

- Benefit: 2 order of magnitude larger time step to reach compression level of 0.1%

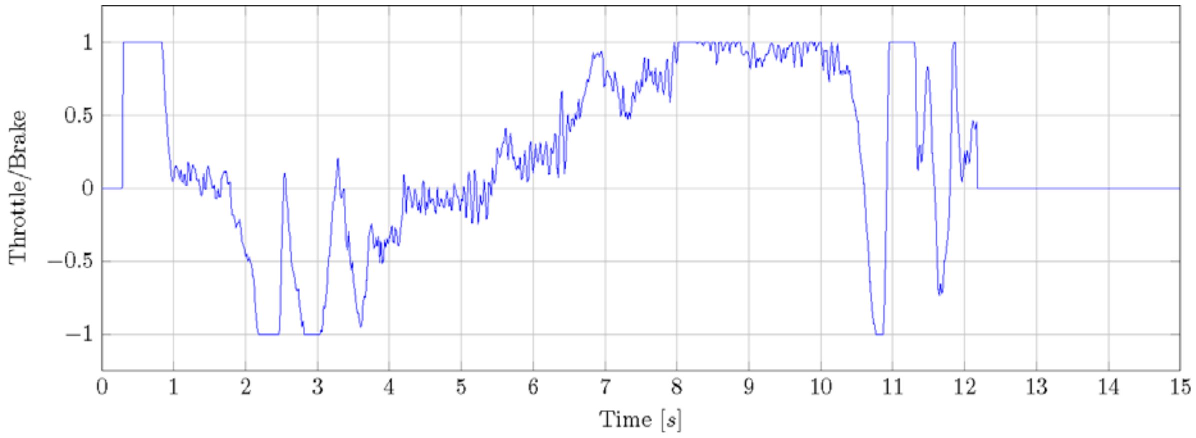


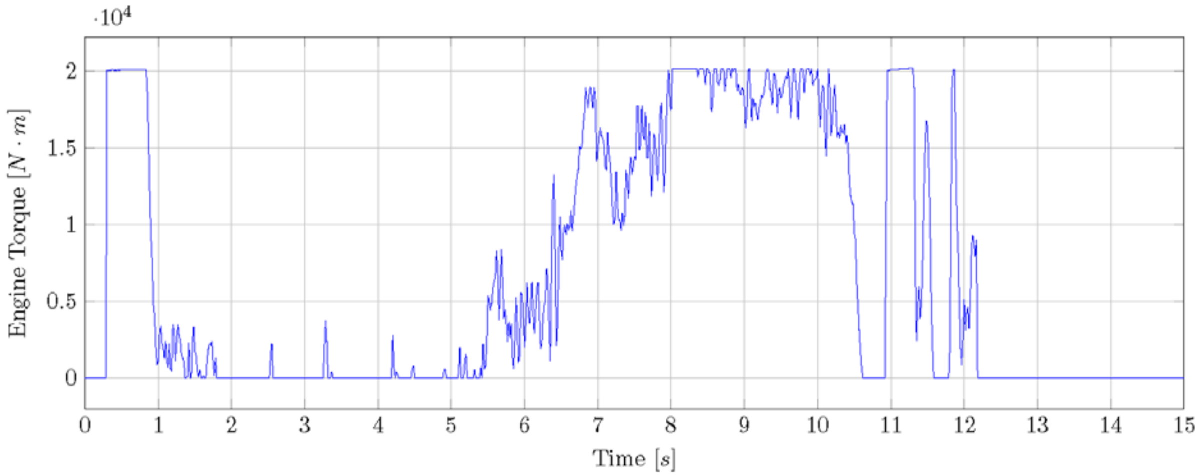
Tracked Vehicle Fording
[chassis z-location and chassis speed]





Tracked Vehicle Fording [throttle/brake & engine torque]





The ALE Viewpoint:
Elasto-Plastic Material via MPM

The Material Point Method (MPM)

- An Arbitrary Lagrangian-Eulerian (ALE) method
  - Similar to Particle In Cell (PIC) and Fluid Implicit Particle (FLIP)
  - Capable of simulating both fluid and solid materials

- Field unknowns stored on markers, similar to SPH

- Background grid used as scratchpad to perform computations
  - Quantities are interpolated from markers onto grid
  - Interpolation performed using cubic B-spline kernel

Stomakhin, A., Schroeder, C., Chai, L., Teran, J., and Selle, A. 2013, “A material point method for snow simulation”. *ACM TOG*

Split Pressure and Shear

- The total stress can be computed as

$$\sigma = -p\mathbf{I} + \mathbf{s}$$

- Split into a pressure dilational (pressure) component and a deviatoric (shear) component

$$\Psi(F_E, F_P) = \underbrace{\mu(F_P)||F_E - R_E||_F^2}_{\text{Deviatoric}} + \underbrace{\frac{\lambda(F_P)}{2}(J_E - 1)^2}_{\text{Dilational}}$$

- The main idea:
  - Solve for pressure using constraint fluids
  - Solve for shear forces using MPM

Stomakhin, A., Schroeder, C., Jiang, C., Chai, L., Teran, J., and Selle, A.,"Augmented MPM for phase-change and varied materials". *ACM TOG 2014*

Coupling Constraint Fluids and MPM

- Stress is split into its two components

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\sigma}_\mu$$

$$\rho \frac{D\boldsymbol{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{g} = \overbrace{\nabla \cdot \boldsymbol{\sigma}_\mu}^{\text{Shear}} - \overbrace{\nabla \boldsymbol{p}}^{\text{Pressure}} + \rho \boldsymbol{g}$$

- Discretized using a Chorin style splitting using intermediate velocity $\boldsymbol{v}^*$

$$\frac{\boldsymbol{v}^* - \boldsymbol{v}^n}{\Delta t} = \frac{1}{\rho^n} \nabla \cdot \boldsymbol{\sigma}_\mu \qquad \rightarrow \text{MPM}$$

$$\frac{\boldsymbol{v}^{n+1} - \boldsymbol{v}^*}{\Delta t} = -\frac{1}{\rho^n} \nabla p^{n+1} + \boldsymbol{g} \qquad \rightarrow \text{Constraint Fluids}$$

Solution Procedure

- First solve for $\boldsymbol{v}^*$ using MPM on grid and map from grid to markers

$$\left(\boldsymbol{I} + \Delta t^2 \boldsymbol{M}^{-1}\frac{\partial^2 \Phi^n}{\partial \hat{x}^2}\right)\boldsymbol{v}^* = \boldsymbol{v}^n - \Delta t \boldsymbol{M}^{-1}\frac{\partial \Phi^n}{\partial \hat{x}}$$

- Then solve EOM to enforce contact and density constraints

$$\boldsymbol{M}(\boldsymbol{v}^{n+1} - \boldsymbol{v}^*) = \underbrace{\Delta t \boldsymbol{f}(t, \boldsymbol{x}^n, \boldsymbol{v}^*)}_{\text{Applied impulse}} + \underbrace{\boldsymbol{D}^f(\boldsymbol{x}^n, t)\gamma^f}_{\text{Density Impulse}} - \underbrace{\boldsymbol{D}^{fb}(\boldsymbol{x}^n, t)\gamma^{fb}}_{\text{Contact Impulse}}$$

$$-\boldsymbol{D}^{f^T}\boldsymbol{v}^{n+1} = \underbrace{\frac{1}{\Delta t}\boldsymbol{C}^f(\boldsymbol{x}^n, t)}_{\text{Density Stabilization term}}$$

$$i \in \mathcal{B}(x^n, \delta) : 0 \leq \underbrace{\frac{1}{\Delta t}C_i^{fb}(\boldsymbol{x}^n)}_{\text{Contact Stabilization term}} + \mathbf{D}_{i,n}^{fb\,T}\boldsymbol{v}^{n+1} - \underbrace{\mu_i\sqrt{\left(\mathbf{D}_{i,u}^{fb\,T}\boldsymbol{v}^{n+1}\right)^2 + \left(\mathbf{D}_{i,w}^{fb\,T}\boldsymbol{v}^{n+1}\right)^2}}_{\text{Relaxation/Tilting Term}} \perp \gamma_{i,n}^{fb} \geq 0$$

$$\left(\gamma_{i,u}^{fb}, \gamma_{i,w}^{fb}\right) = \underset{\sqrt{(\gamma_{i,u}^{fb})^2 + (\gamma_{i,w}^{fb})^2} \leq \mu_i \gamma_{i,n}^{fb}}{\operatorname{argmin}} \boldsymbol{v}^T\left(\gamma_{i,u}^{fb}\mathbf{D}_{i,u}^{fb} + \gamma_{i,w}^{fb}\mathbf{D}_{i,w}^{fb}\right)$$

The Cone Complementarity Problem (CCP)

Contacts:
- Rigid-rigid
- Fluid-rigid
- Tetrahedral-rigid

Constraints
- Joints
- Density
- Tetrahedron

Conic constraint for friction

Find :

$$(\gamma_i^c)^{n+1} \quad \text{s.t.} \quad \Upsilon_i \ni (\gamma_i^c)^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_i \in \Upsilon_i^\circ$$

$$(\gamma_j^{fb})^{n+1} \quad \text{s.t.} \quad \Upsilon_j \ni (\gamma_j^{fb})^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_j \in \Upsilon_j^\circ$$

$$(\gamma_k^{tb})^{n+1} \quad \text{s.t.} \quad \Upsilon_k \ni (\gamma_k^{tb})^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_k \in \Upsilon_k^\circ$$

$$(\gamma_l^j)^{n+1} \quad \text{s.t.} \quad \mathbb{R}^m \ni (\gamma_l^j)^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_l \in \{0\}^{n_j}$$

$$(\gamma_m^f)^{n+1} \quad \text{s.t.} \quad \mathbb{R}^m \ni (\gamma_m^f)^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_m \in \{0\}^{n_f}$$

$$(\gamma_o^t)^{n+1} \quad \text{s.t.} \quad \mathbb{R}^m \ni (\gamma_o^t)^{n+1} \perp -\left(\boldsymbol{N}\boldsymbol{\gamma}^{n+1} + \boldsymbol{r}\right)_o \in \{0\}^{n_t}$$

$$\text{where} \quad a \in \{i, j, k\}, \quad [x, y, z]^T \in \mathbb{R}^3$$

$$\Upsilon_a = \sqrt{y^2 + z^2} \leq \mu_a^f x$$

$$\text{and} \quad \Upsilon_a^\circ = x \leq -\mu_a^f \sqrt{y^2 + z^2}$$

Optimization Formulation

- CCP leads to quadratic problem with conic constraints

$$\min f(\boldsymbol{\gamma}) = \frac{1}{2}\boldsymbol{\gamma}^T \boldsymbol{N}\boldsymbol{\gamma} + \boldsymbol{r}^T\boldsymbol{\gamma}$$

$$\text{subject to } \boldsymbol{\gamma}^{[c,fb,tb]} \in \Upsilon,$$

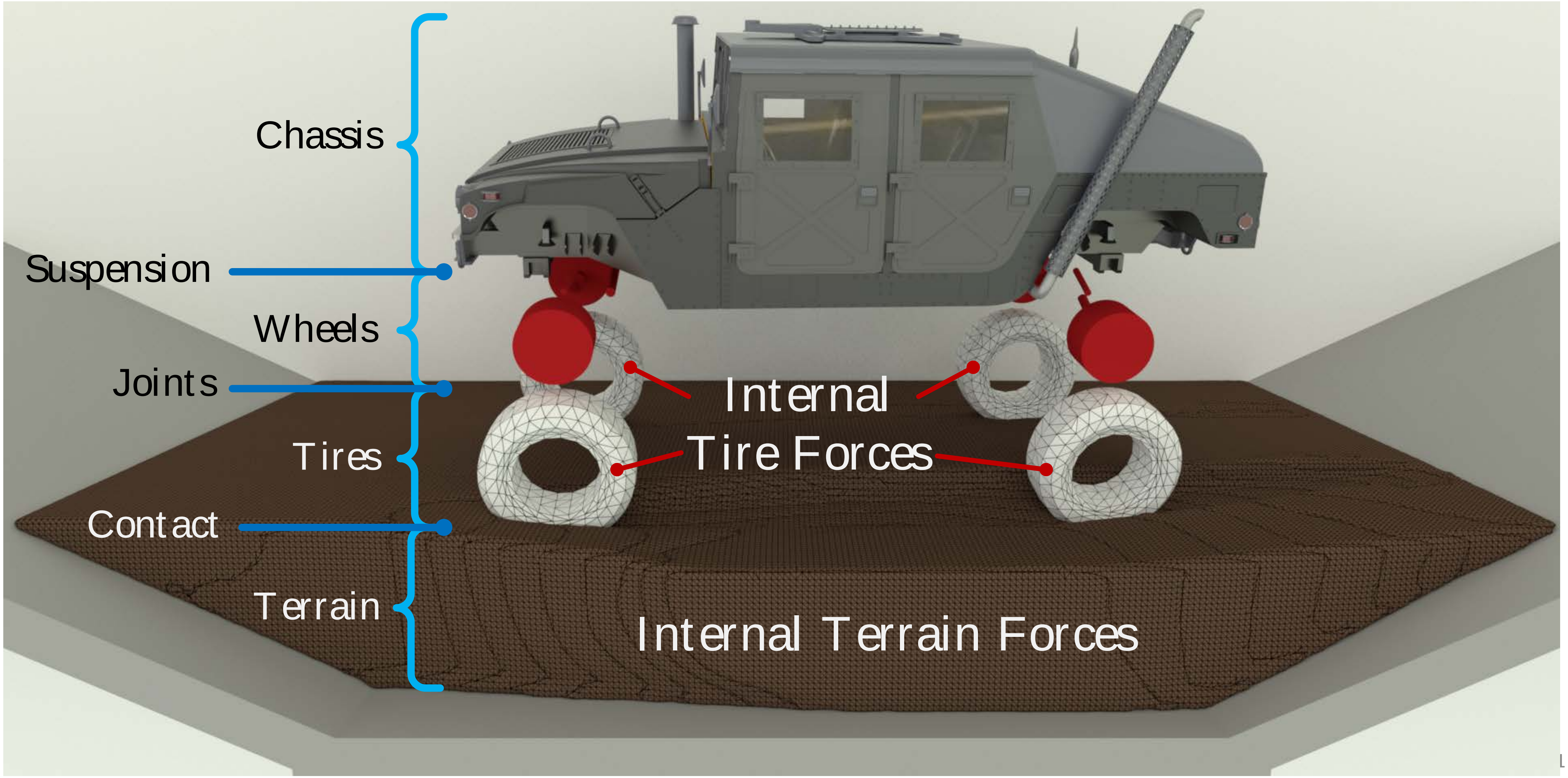
$$\boldsymbol{\gamma}^{[j,f,t]} \in \mathbb{R}^n$$

"Multi-Physics Computational Dynamics Using Complementarity and Hybrid Lagrangian-Eulerian Methods," H. Mazhar, PhD Thesis, UW-Madison, 2016

Putting it all together

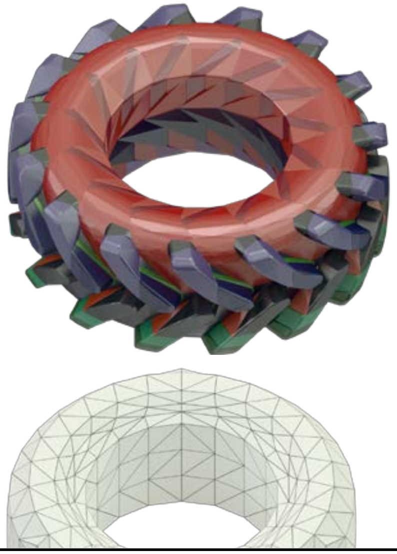
Meshless approach to terrain deformation

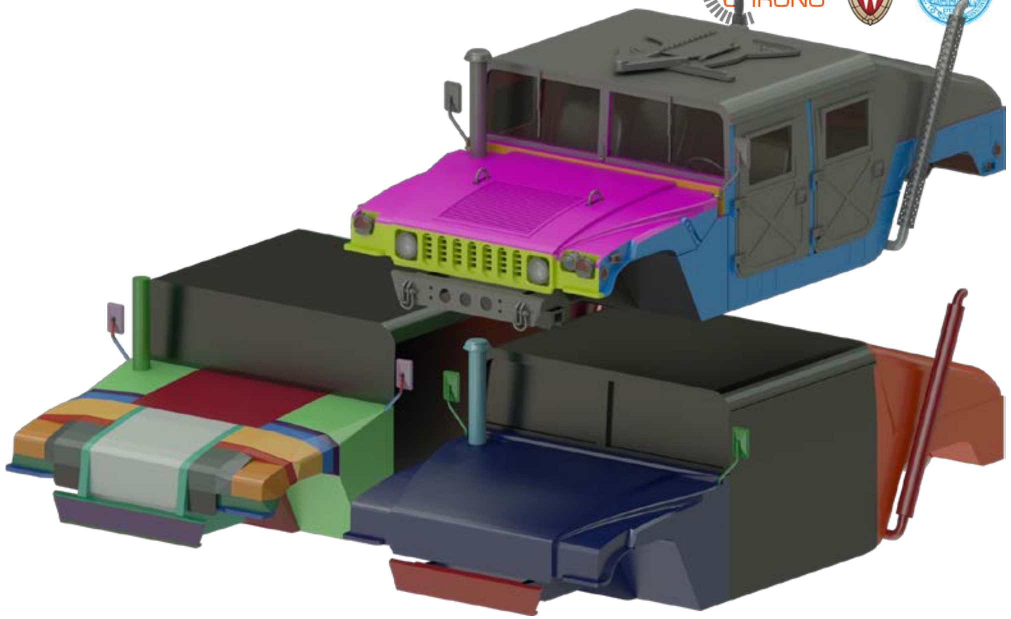
Mud + Deformable Tire + HMMWV



Vehicle Terrain Interaction

- HMMWC Specifications:
  - Mass: 2086 kg
  - Target velocity: 2m/s
  - Double wishbone suspension
  - 4WD powertrain





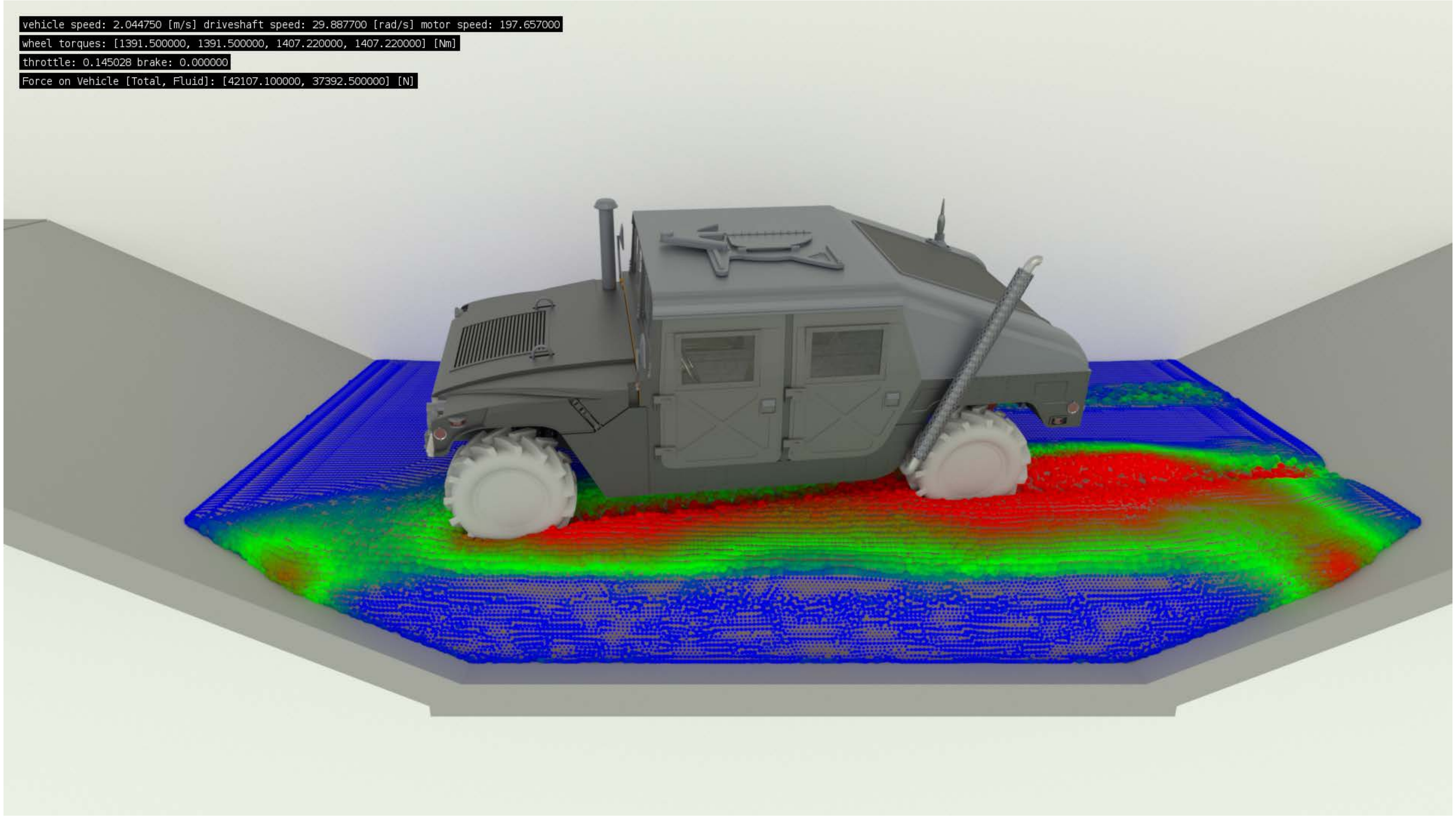
5.18m (17 ft)

Bottom Length:
4.57m (15 ft)

Vehicle on Mud: Constraint Fluids

Continuum Terrain	
N	1 426 663
m	0.023 $[kg]$
ρ	1200 $[kg/m^3]$
h	0.032 $[m]$
E	$7 \times 10^5 [Pa]$
ν	.2
θ_c	2.5e-2
θ_s	2.5e-2
Simulation	
Δt	0.001 $[s]$
Solver	1000 Bilateral 50 Full
Sim. Length	12 sec
Time	30 Hours

Displacement of the soil elements



Looking Ahead, Chrono Multi-Physics

- Enhance multi-physics
- Constitutive models (mud/slurry)
- Validation
- Robustness
- Speed

Validation Status

- Extensive validation for granular material, both Complementarity and Penalty
  - Shear, triaxial, cone penetration, ball drop, rate of flow
- Some validation for fluid-solid interaction
  - Incompressibility test, sloshing, Poiseuille flow, flow in pipes, dam break
- No validation for mud/slit/snow simulations