
Basics of Vehicle Dynamics

INDEX

<i>Chapter</i>	<i>Description</i>
H1	Introduction to vehicle dynamics <ul style="list-style-type: none">• Basics• Handling analysis
H2	Tyre behaviour <ul style="list-style-type: none">• Forces generated by tyre• Tyre effects• Tyre transient response• Handling tyre models
H3	Suspension characteristics <ul style="list-style-type: none">• Geometrical data• Elasto-kinematics analysis• Suspension typologies and their characteristics• Suspension model
H4	Steering systems characteristics <ul style="list-style-type: none">• King pin axis geometry• Ackerman error• Steering wheel systems
H5	Evaluation criteria of handling and steering systems <ul style="list-style-type: none">• Testing manoeuvres• Quality indexes

**Basics of
Vehicle Dynamics**

Module H1

Introduction to Vehicle Dynamics

- Basics
 - Handling analysis
-

Basics

VEHICLE DYNAMICS STUDY

GOALS:

❖ **Vehicles behaviour analysis and improvement in terms of:**

- Vehicle directional control stability
- Roadholding
- Steerability
- Maneuverability

❖ **ACTIVE SAFETY Increase**

active/passive

❖ **Better users pleasure**

comfort, steering, handling, acceleration

*p. stability: control of motion in all
 • comfort: stability, hydrostatic (performance)
 (performance)*

VEHICLE DYNAMICS

STUDIES VEHICLE MOTION WHEN
EXTERNAL FORCES ARE APPLIED

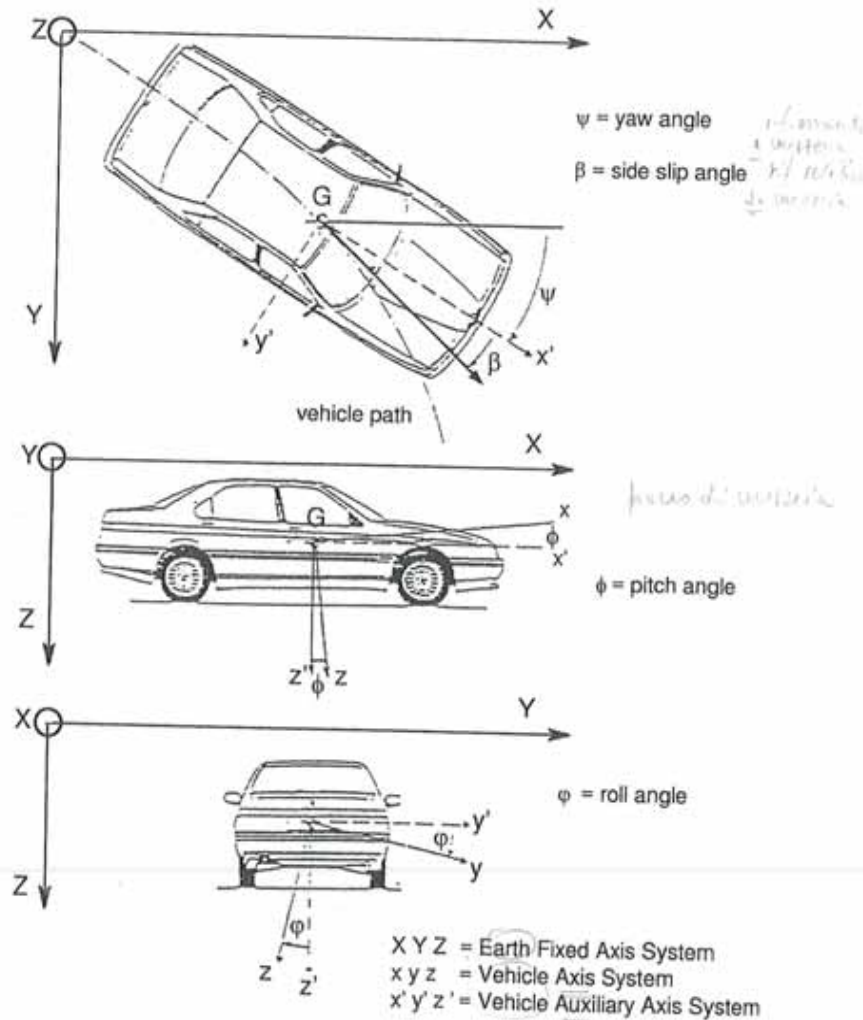
FORCES APPLIED TO THE VEHICLE:

- ❖ Ground-Tyre Forces
- ❖ Aerodynamic forces on vehicle due to relative air motion *quite large!*
- ❖ Gravitational force component



(In a system)
note? (force system)
here:
force
differentiable
analyse
discrete field

VEHICLE AXIS SYSTEMS AND VEHICLE MOTIONS



VEHICLE AXIS SYSTEMS AND VEHICLE MOTIONS

- ❖ Sprung mass: All mass which is supported by the suspension, including portions of the mass of the suspension members
- ❖ Unsprung mass: All mass which is not carried by the suspension system, but is supported by tyre or wheel, and considered to move with it
- ❖ Three basic axis systems:
 - Earth-Fixed Axis System XYZ
 - Vehicle Axis System xyz (fixed in the vehicle body and moving with it)
 - Vehicle Auxiliary Axis System $x'y'z'$
- ❖ Lateral Acceleration A_y : is the component of the vector acceleration of the vehicle CG parallel to the y' direction *vehicle system*
- ❖ Yaw Angle ψ : is the angle between the Earth-Fixed X -axis and the vehicle auxiliary x' -axis
- ❖ Sideslip Angle β : is the angle between the vehicle auxiliary x' -axis and the vehicle velocity vector
- ❖ Vehicle Pitch Angle ϕ : is the angle between the vehicle x -axis and the vehicle auxiliary x' -axis
- ❖ Vehicle Roll Angle θ : is the angle between the vehicle y -axis and the vehicle auxiliary y' -axis

Handling analysis

HANDLING ANALYSIS : motion equations statement

STEADY STATE MANOEUVRES

no driver skills - open-loop
STEERING PAD
(quasi-) Visibeaming

- CONSTANT PATH RADIUS
- CONSTANT SPEED
(radius is changing - I turn)

NON LINEAR ALGEBRIC EQUATIONS SYSTEM

quasi-linear?

TRANSIENT STATE MANOEUVRES

driver is important
DRIVER INPUT TIME HISTORIES
ARE IMPOSED

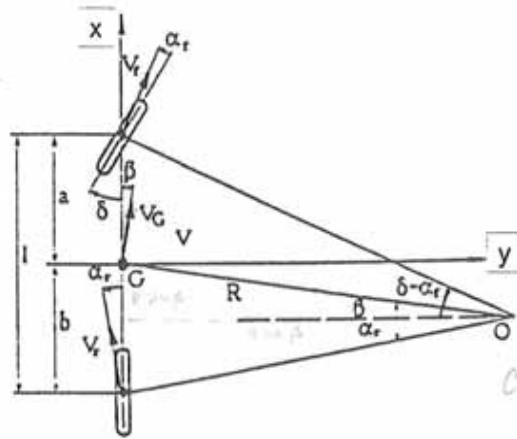
Eg. STEP STEERING INPUT,
TIP OUT

NON LINEAR DIFFERENTIAL EQUATIONS SYSTEM

time dependent

"BICYCLE MODEL"

- No lateral and longitudinal load transfer
- No rolling or pitching motion
- No chassis or suspension compliance effects
- No aerodynamic effects
- Position control (front wheel angle)
- Linear range tyres



STEADY STATE CORNERING

$\frac{b - R \sin \beta}{R \cos \beta} = \tan \alpha_r$ if $\beta, \alpha_r \rightarrow 0$ then $\frac{b}{R} - \beta \approx \alpha_r$

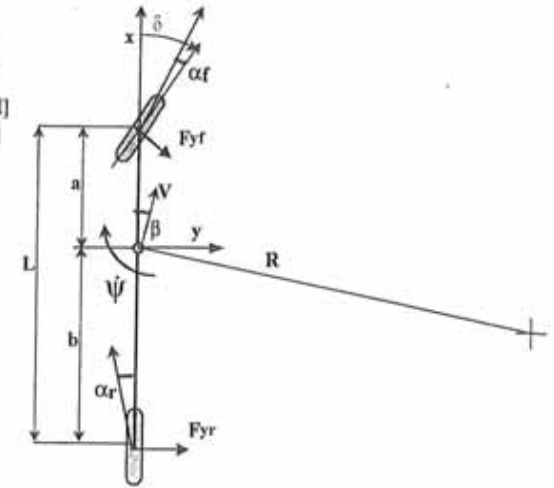
$\beta \approx \frac{b}{R} - \alpha_r$

$\frac{a + R \sin \beta}{R \cos \beta} = \tan(\delta - \alpha_f)$ if $\beta, \alpha_f \rightarrow 0$ then $\frac{a}{R} + \frac{b}{R} - \alpha_r = \delta - \alpha_f$

$\delta \approx \frac{L}{R} + \alpha_f - \alpha_r$

YAW EQUILIBRIUM

- F_{yf} : lateral cornering force at front axle [N]
- F_{yr} : lateral cornering force at rear axle [N]
- $C_{\alpha f}$: Cornering stiffness at front axle [N/rad]
- $C_{\alpha r}$: Cornering stiffness at rear axle [N/rad]
- m : Mass of the vehicle [kg]
- V : Forward velocity [m/s]
- R : Radius of the turn [m]
- a : longitudinal distance between front axle and center of gravity [m]
- δ : Steer angle [rad]
- δ_{vol} : Steering wheel angle [rad]
- τ : Steering ratio



$(\delta \approx 0 \implies \cos \delta \approx 1)$

Lateral direction: $\sum F_y = F_{yf} + F_{yr} = F_c$

About the center of gravity: $\sum M_z = F_{yf} \cdot a - F_{yr} \cdot b = 0$

where

$F_c = m \cdot a_y = m \cdot \dot{\psi}^2 \cdot R = m \cdot \frac{V^2}{R}$

Solving the cornering equations yields:

$F_{yf} = m \cdot \frac{V^2}{R} \cdot \frac{b}{L} = m_f \cdot a_y$

$F_{yr} = m \cdot \frac{V^2}{R} \cdot \frac{a}{L} = m_r \cdot a_y$

(YAW EQUILIBRIUM)

Supposing :

$$F_y = C \cdot \alpha$$

modello potenza lineare

$$\alpha_f = \frac{F_{yf}}{C_f} = \frac{m_f \cdot V^2}{C_f \cdot R} \quad \alpha_r = \frac{F_{yr}}{C_r} = \frac{m_r \cdot V^2}{C_r \cdot R}$$

$\psi = \frac{V_{steer}}{r}$
 $\propto V^2$
 speed dependent!

$$\delta_{vol} = \frac{L}{R} \cdot \tau - (\alpha_f - \alpha_r) \cdot \tau = \frac{L}{R} \cdot \tau + \left(\frac{m_f}{C_f} - \frac{m_r}{C_r} \right) \cdot \frac{V^2}{R} \cdot \tau = \delta_{vol0} + KUS \cdot a_y$$

$\delta_{vol} = \delta_{vol0} + KUS \cdot a_y$
 val costante in direzione
 tire data
 ay dependent

$$KUS = \left(\frac{m_f}{C_f} - \frac{m_r}{C_r} \right) \cdot \tau \Rightarrow \text{Understeer gradient [rad/m/s}^2]$$

$$\beta = \frac{b}{R} - \alpha_r = \frac{b}{R} - \frac{m_r}{C_r} \cdot \frac{V^2}{R} = \beta_0 - KBETA \cdot a_y$$

$$KBETA = \frac{m_r}{C_r} \Rightarrow \text{Sideslip angle gradient [rad/m/s}^2]$$

$$\beta = \frac{b}{R} - \frac{m_r}{C_r} \cdot \frac{V^2}{R} = \frac{b}{V^2} \cdot a_y - \frac{m_r}{C_r} \cdot a_y = \left(\frac{b}{V^2} - \frac{m_r}{C_r} \right) \cdot a_y$$

monotone decrescente

$$\left(\frac{b}{V^2} - \frac{m_r}{C_r} \right) = 0 \quad V = \sqrt{\frac{C_r \cdot b}{m_r}}$$

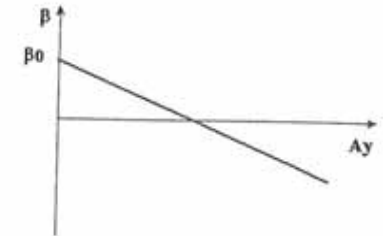
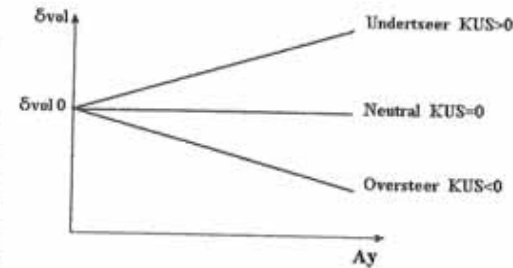
Speed at which $\beta=0$

da > alla velo hanno velo > 0
 < 0 alla velo < 0

C.12 ovatta !!

(YAW EQUILIBRIUM)

•Steering pad constant radius

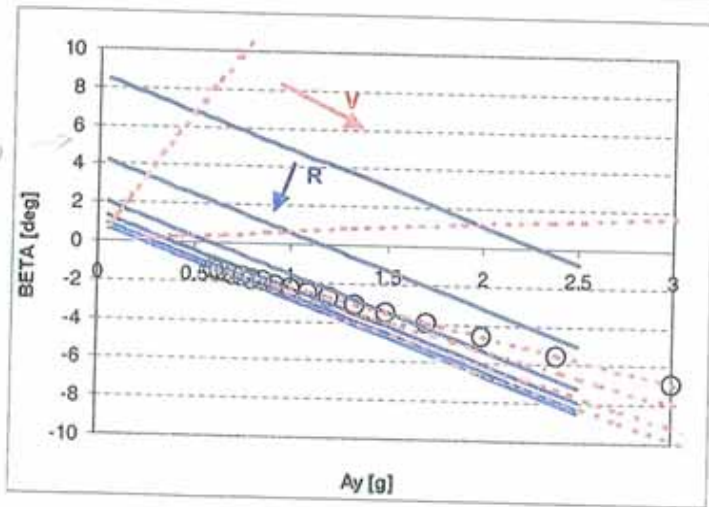
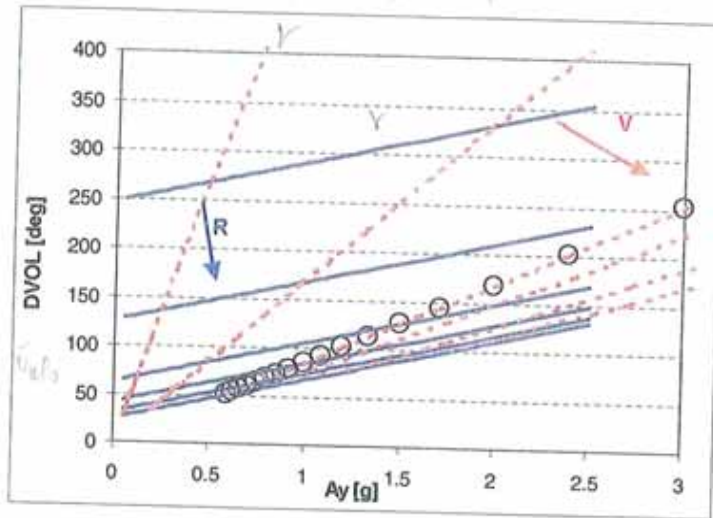


Lateral acceleration gain : $\frac{a_y}{\delta_{vol}} = \frac{1}{\frac{L}{V^2} \cdot \tau + KUS}$

Yaw velocity gain : $\frac{\dot{\psi}}{\delta_{vol}} = \frac{1}{\frac{L}{V} \cdot \tau + KUS \cdot V}$

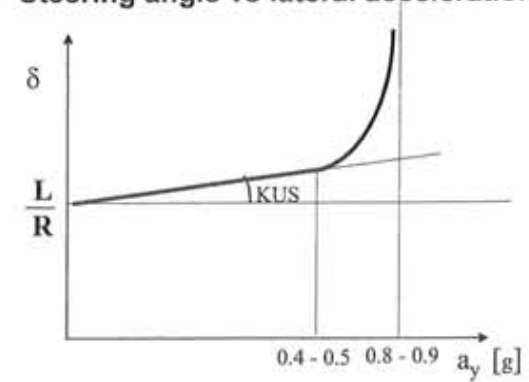
(YAW EQUILIBRIUM:)

• UNDERSTEER VEHICLE

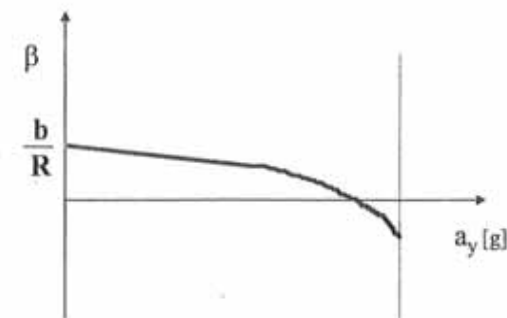


(YAW EQUILIBRIUM: NON LINEAR CASE)

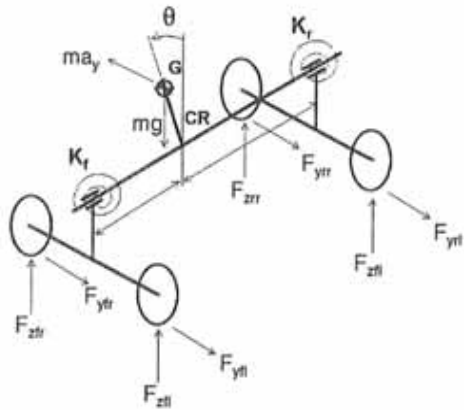
- Steering angle vs lateral acceleration



- Sideslip angle vs lateral acceleration



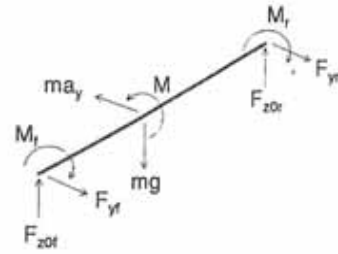
ROLL EQUILIBRIUM



h_{CR} = roll centre height

h_G = mass centre height

$\Delta h = h_G - h_{CR}$



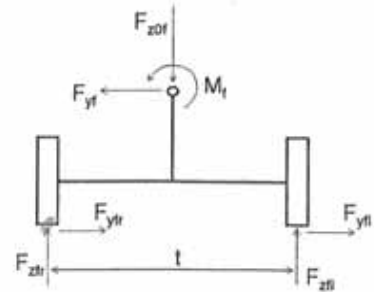
$$\begin{cases} F_{z0f} = m \cdot g \cdot \frac{b}{a+b} \\ F_{z0r} = m \cdot g \cdot \frac{a}{a+b} \end{cases}$$

$$\begin{cases} F_{yf} = m \cdot a_y \cdot \frac{b}{a+b} \\ F_{yr} = m \cdot a_y \cdot \frac{a}{a+b} \end{cases}$$

just suppose that: $\theta \approx 0$

$M = M_f + M_r = (K_f + K_r) \cdot \theta = m \cdot a_y \cdot \Delta h$

$\theta = \frac{m \cdot a_y \cdot \Delta h}{K_f + K_r}$



$F_{z0f} \cdot t + F_{yf} \cdot h_{CR} + M_f - F_{z0f} \cdot \frac{t}{2} = 0$

$F_{z0f} = \frac{1}{t} \left[F_{z0f} \cdot \frac{t}{2} - M_f - F_{yf} \cdot h_{CR} \right]$

$F_{z0f} = \frac{1}{t} \left[F_{z0f} \cdot \frac{t}{2} - K_f \cdot \theta - F_{yf} \cdot h_{CR} \right]$

(ROLL EQUILIBRIUM)

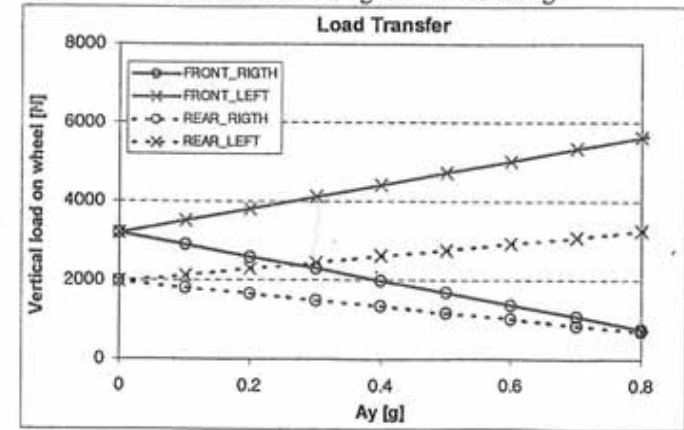
$$\begin{cases} F_{z0f} = \frac{m \cdot g}{2} \cdot \frac{b}{a+b} - m \cdot a_y \cdot \left(\frac{K_f}{K_f + K_r} \cdot \frac{\Delta h}{t} + \frac{b}{a+b} \cdot \frac{h_{CR}}{t} \right) \\ F_{z0r} = \frac{m \cdot g}{2} \cdot \frac{b}{a+b} + m \cdot a_y \cdot \left(\frac{K_f}{K_f + K_r} \cdot \frac{\Delta h}{t} + \frac{b}{a+b} \cdot \frac{h_{CR}}{t} \right) \\ F_{z0l} = \frac{m \cdot g}{2} \cdot \frac{a}{a+b} - m \cdot a_y \cdot \left(\frac{K_r}{K_f + K_r} \cdot \frac{\Delta h}{t} + \frac{a}{a+b} \cdot \frac{h_{CR}}{t} \right) \\ F_{z0r} = \frac{m \cdot g}{2} \cdot \frac{a}{a+b} + m \cdot a_y \cdot \left(\frac{K_r}{K_f + K_r} \cdot \frac{\Delta h}{t} + \frac{a}{a+b} \cdot \frac{h_{CR}}{t} \right) \end{cases}$$

Load transfer on single wheels

$$\begin{cases} F_{z0f} + F_{z0r} = \frac{m \cdot g}{2} + m \cdot a_y \cdot \frac{h_G}{t} \\ F_{z0l} + F_{z0r} = \frac{m \cdot g}{2} - m \cdot a_y \cdot \frac{h_G}{t} \end{cases}$$

Load transfer on the right and left side

•Constant radius right hand cornering



TRANSIENT BEHAVIOUR

Motion equation: $\bar{F} = \frac{d\bar{L}}{dt}$, $\bar{M} = \frac{d\bar{H}}{dt}$

$\bar{L} = m \cdot \bar{V}$, $\bar{H} = J_z \cdot \dot{\psi}$

$\left(\frac{d\bar{A}}{dt} \right)_i = \left(\frac{d\bar{A}}{dt} \right)_m + \bar{\omega} \wedge \bar{A}$

$\bar{F} = \frac{d\bar{L}}{dt} = \frac{d(m \cdot \bar{V})}{dt} + \bar{\psi} \wedge \bar{L}$

$\bar{F} = \frac{d\bar{L}}{dt} = F_x \cdot \bar{i} + F_y \cdot \bar{j} = m \cdot \left(\frac{dV_x}{dt} \bar{i} + \frac{dV_y}{dt} \bar{j} \right) - (m \cdot V_y \cdot \dot{\psi} \cdot \bar{i} - m \cdot V_x \cdot \dot{\psi} \cdot \bar{j})$

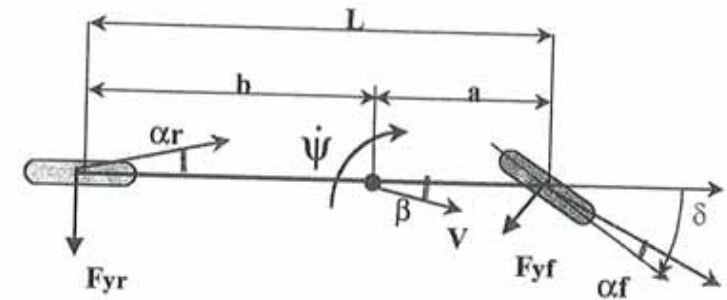
just suppose that: $V_x = V$, $V_y = V \cdot \beta$

$\sum F_x = m \cdot (\dot{V} - V \cdot \dot{\psi} \cdot \beta) = m \cdot a_x$
 $\sum F_y = m \cdot (\dot{V} \cdot \beta + V \cdot \dot{\beta} + V \cdot \dot{\psi}) = m \cdot a_y$

$\bar{M} = \frac{d\bar{H}}{dt} = \frac{d(J_z \cdot \dot{\psi})}{dt} + \bar{\psi} \wedge \bar{H} = J_z \cdot \ddot{\psi}$ ($\bar{\psi} \wedge \bar{H} = 0$)

$\sum M_z = J_z \cdot \ddot{\psi}$

Lateral forces equilibrium and yaw equilibrium



$F_{yr} + F_{yf} \cdot \cos \delta = m \cdot (\dot{V} \cdot \beta + V \cdot \dot{\beta} - V \cdot \dot{\psi})$
 $-b \cdot F_{yr} + a \cdot F_{yf} \cdot \cos \delta = J_z \cdot \ddot{\psi}$

$F_{yf} = C_f \cdot \alpha_f$

$F_{yr} = C_r \cdot \alpha_r$

$\alpha_f = \delta - \beta - \frac{\dot{\psi} \cdot a}{V}$

$\alpha_r = -\beta + \frac{\dot{\psi} \cdot b}{V}$

$V = \text{Constant} \implies \dot{V} = 0$

$\delta \sim 0 \implies \cos \delta \sim 1$

$C_r \cdot \alpha_r + C_f \cdot \alpha_f = m \cdot (V \cdot \dot{\beta} - V \cdot \dot{\psi})$
 $-b \cdot C_r \cdot \alpha_r + a \cdot C_f \cdot \alpha_f = J_z \cdot \ddot{\psi}$

Example:

