Validation of a bilinear, gradient-deficient ANCF shell element in Chrono::FEA

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Abstract This report summarizes the validation efforts for the verifying Chrono's implementation of a bilinear, shear deformable ANCF shell element. For more details on the implementation of finite elements in Chrono, the reader is referred to the white paper library on the website: http://api.chrono.projectchrono.org/whitepaper_root.html

Keywords: Validation, ANCF shell element, nonlinear finite elements, Chrono

Contents

1. Introduction

This brief report details the validation of the bilinear, gradient-deficient ANCF shell element implemented in Chrono for isotropic, orthotropic, and layered materials. Chrono's users may find detailed descriptions of the this laminated ANCF finite element implementation in Refs. [1, 2].

2. ANCF Shell Element

2.1 ANCF Shell Element Kinematics

The basics kinematics of the absolute nodal coordinate formulation (ANCF) shell finite element implemented in Chrono is depicted in Fig.1. The nodal position is defined as a function of the global position and the transverse gradient vector $\frac{\partial \mathbf{r}^i}{\partial z^i}(x^i, y^i)$ which describes the orientation of the cross section. The element's positions and gradients on the mid-plane can be fully described as

$$\mathbf{r}^i_m(x^i, y^i) = \mathbf{S}^i_m(x^i, y^i)\mathbf{e}^i_p, \quad \frac{\partial \mathbf{r}^i}{\partial z^i}(x^i, y^i) = \mathbf{S}^i_m(x^i, y^i)\mathbf{e}^i_g, \tag{1}$$

where $\mathbf{S}^i_m$ is a bilinear shape function matrix, $\mathbf{e}^{ik}_p = \mathbf{r}^{ik}$, and $\mathbf{e}^{ik}_g = \partial \mathbf{r}^{ik}/\partial z^i$. The shape function matrix is arranged as $\mathbf{S}^i_m = [S^i_1\mathbf{I} \;\; S^i_2\mathbf{I} \;\; S^i_3\mathbf{I} \;\; S^i_4\mathbf{I}]$. The bilinear shape functions of the ANCF shell element are given by the following expressions

$$S^i_1 = \frac{1}{4}(1-\xi^i)(1-\eta^i), S^i_2 = \frac{1}{4}(1+\xi^i)(1-\eta^i),$$
$$S^i_3 = \frac{1}{4}(1+\xi^i)(1+\eta^i), \text{ and } S^i_4 = \frac{1}{4}(1-\xi^i)(1+\eta^i).$$

The position of an arbitrary point in the shell may be described as

$$\mathbf{r}^i(x^i, y^i, z^i) = \mathbf{S}^i(x^i, y^i, z^i)\mathbf{e}^i, \tag{2}$$

where the combined shape function matrix is given by $\mathbf{S}^i = [\mathbf{S}^i_m \;\; z^i\mathbf{S}^i_m]$. Similarly, the coordinates of the element may be grouped as $\mathbf{e}^i = [(\mathbf{e}^i_p)^T \;\; (\mathbf{e}^i_g)^T]^T$. Note that Eq. (2) incorporates the local coordinate along the element thickness. Based on this kinematic description of the shell element, the Green-Lagrange strain tensor may be calculated as

$$\mathbf{E}^i = \frac{1}{2}\left(\left(\mathbf{F}^i\right)^T\mathbf{F}^i - \mathbf{I}\right), \tag{3}$$

where $\mathbf{F}^i$ is the deformation gradient matrix defined as the current configuration over the reference configuration. Using the current absolute nodal coordinates, this matrix may be defined as

$$\mathbf{F}^i = \frac{\partial \mathbf{r}^i}{\partial \mathbf{X}^i} = \frac{\partial \mathbf{r}^i}{\partial \mathbf{x}^i}\left(\frac{\partial \mathbf{X}^i}{\partial \mathbf{x}^i}\right)^{-1}. \tag{4}$$

The strain tensor can then expressed in vector form in the following manner

$$\boldsymbol{\varepsilon}^i = \begin{bmatrix}\varepsilon^i_{xx} & \varepsilon^i_{yy} & \gamma^i_{xy} & \varepsilon^i_{zz} & \gamma^i_{xz} & \gamma^i_{yz}\end{bmatrix}^T, \tag{5}$$

where $\boldsymbol{\varepsilon}^i$ is the engineering strain vector at the deformed configuration. The internal force vector is then calculated from the second Piola-Kirchhoff stress, $\boldsymbol{\sigma}^i$, and the derivative of the engineering strain with respect to the nodal coordinates. The elastic internal forces are spatially integrated over the element volume using Gaussian quadrature:

$$\mathbf{Q}^i_k = \int_{V^i_0}\left(\frac{\partial \varepsilon^i}{\partial \mathbf{e}^i}\right)^T \sigma^i \mathrm{d}V^i_0 \tag{6}$$

Chrono's implementation includes some numerical techniques that guarantee proper convergence of this finite element, thereby avoiding kinematic locking. These may be summarized as follows

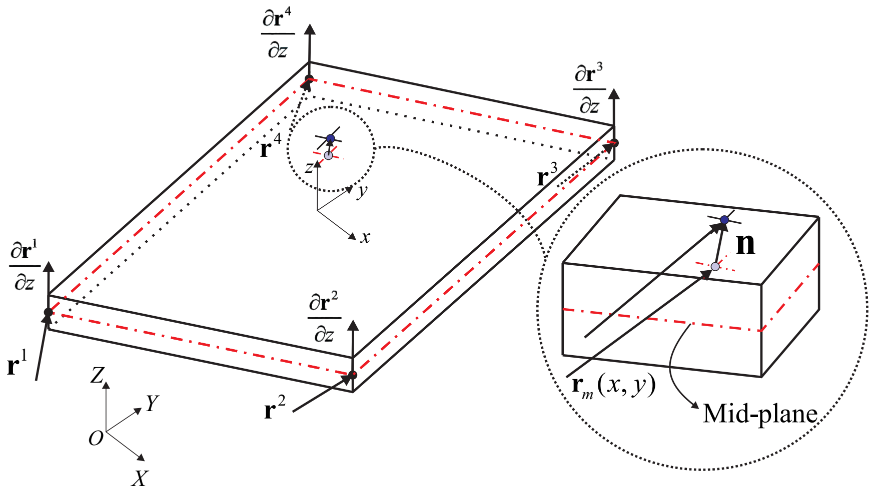

Figure 1: ANCF shell element's kinematic description. This finite element employs bilinear shape functions to describe mid-plane deformation and has a position vector and a gradient vector normal to the surface as nodal coordinates.

- Enhanced Assumed Strain (EAS). This technique alleviates transverse shear locking and is implemented as an inner Newton-Raphson loop in the elastic forces which enhances the compatible strain.
- Assumed Natural Strain (ANS). This technique modifies the interpolation of shear and thickness compatible strains to avoid locking effects.

3. Laminated Composite Material Formulation

Chrono implements an orthotropic Saint-Venant-Kirchhoff material model with a material moduli matrix defined as:

$$C^{ijkl} = \frac{\partial^2 W}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}, \tag{7}$$

where each elastic coefficient depends on the fiber direction. This tensor's elements may be rewritten in the following fashion

$$C^{ijkl} = \left(\mathbf{b}^i \cdot \mathbf{a}_b\right)\left(\mathbf{b}^j \cdot \mathbf{a}_b\right)\left(\mathbf{b}^k \cdot \mathbf{a}_c\right)\left(\mathbf{b}^l \cdot \mathbf{a}_d\right)\bar{C}^{abcd}, \tag{8}$$

where $\bar{C}^{abcd}$ is the tangent material moduli defined in the fiber coordinate system $\left[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3\right]$. The direction of the fiber is assumed to be described by direction $\mathbf{a}_1$. The matrix of tangential material moduli is defined by

$$\left[\bar{C}^{ijkl}\right] = \begin{bmatrix} \bar{C}^{1111} & \bar{C}^{1122} & 0 & \bar{C}^{1133} & 0 & 0 \\ \bar{C}^{1122} & \bar{C}^{2222} & 0 & \bar{C}^{2233} & 0 & 0 \\ 0 & 0 & \bar{C}^{1212} & 0 & 0 & 0 \\ \bar{C}^{1133} & \bar{C}^{2233} & 0 & \bar{C}^{3333} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{C}^{2323} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{C}^{1313} \end{bmatrix}. \tag{9}$$

The internal elastic forces and element mass matrix are then calculated using the adjusted material moduli at each lamina layer for a total of N layers by integrating over the element volume. The generalized elastic and inertia forces are defined, respectively, by the following equations

$$\mathbf{Q}_s^i = -\sum_{k=1}^{N} \int_{V_0^{ik}} \left(\frac{\partial \varepsilon^{ik}}{\partial \mathbf{e}^i} \right)^T \frac{\partial W^{ik}\left(\hat{\varepsilon}^{ik}\right)}{\partial \varepsilon^{ik}} \mathrm{d}V_0^{ik}, \tag{10}$$

$$\mathbf{M}^i = \sum_{k=1}^{N} \int_{V_0^{ik}} \rho_0^{ik} \left(\mathbf{S}^i\right)^T \mathbf{S}^i \mathrm{d}V_0^{ik}, \tag{11}$$

where $W^{ik}(\varepsilon^{ik})$ is the elastic energy generated by Green-Lagrange strains and its assumed natural and enhanced assumed counterparts. Superscripts i and k in Eqs. (10)-(11) refer to the element number and the layer number, respectively. Each layer of the laminated composite shell element is integrated over the thickness using two Gauss points. One example of a composite shell is the "balanced" laminated shell which has two stacked layers whose fibers are oriented along an angle $\pm\theta$ with respect to the local axis x.

4. Validation of the ANCF shell element

In this section we validate the implementation of Chrono 's absolute nodal coordinate shell element using commercial software.

4.1 Validation of the isotropic implementation

This subsection shows some results of the validation of the elastic, isotropic implementation of a four-node shell element implemented in Chrono . This nonlinear finite element uses absolute nodal coordinates to describe the position of the nodes and a vector normal to the shell surface.

4.1.1 Model's definition

The first model chosen to verify Chrono 's implementation is a cantilever flat shell subjected to a sudden, constant load at a corner. Due to the application of a sudden load, the shell undergoes vibrations that are damped out by adding structural damping. The length, width, and thickness of the flat shell (plate) are assumed to be 1.0 m, 1.0 m, and 0.01 m, respectively. The Young modulus and Poisson ratio are $2.1\text{x}10^8$ Pa and 0.3, respectively. A vertical force of 50 N is applied downwards in the absence of a gravity field. Structural damping is purposely employed to bring the system to static equilibrium. More details on this example may be found in Ref. [2].
The details of this model and its corresponding results may be consulted and obtained running the unit test test_ANCFShellIso.cpp, which is available in Chrono 's repository.

4.1.2 Numerical validation

The time evolution for the system and loads defined in the previous subsection results in underdamped oscillations of the shell. Figure 2 displays the oscillations of the shell's loaded tip for two different mesh sizes.

Comparison is performed with the commercial software ANSYS to ensure that Chrono 's implementation converges to the correct displacement and at a good rate. Reference results are obtained from a 64x64 mesh in ANSYS. It must be emphasized that, while ANSYS's results are obtained from static, finite strain results, Chrono performs dynamic simulations to find the static equilibrium position. Convergence results are plotted in Fig. 3.

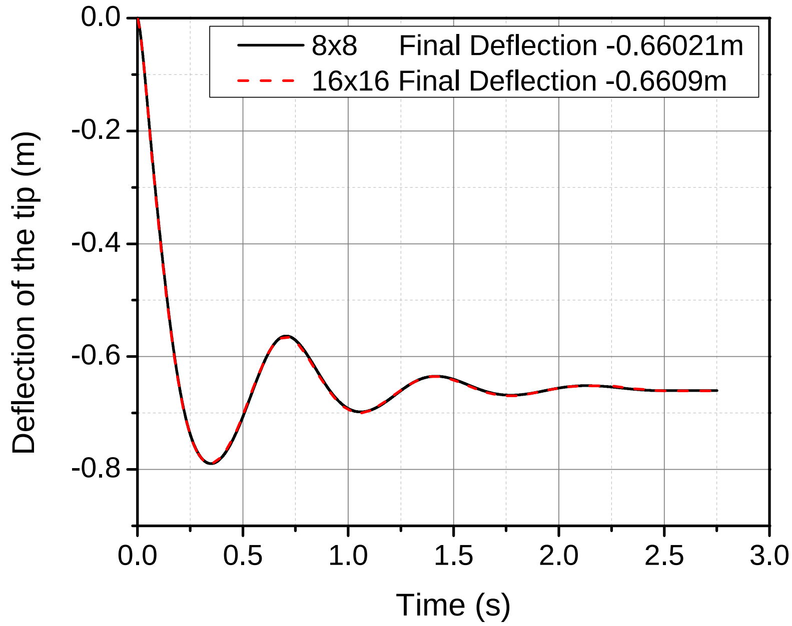

Figure 2: Tip vertical coordinate time evolution

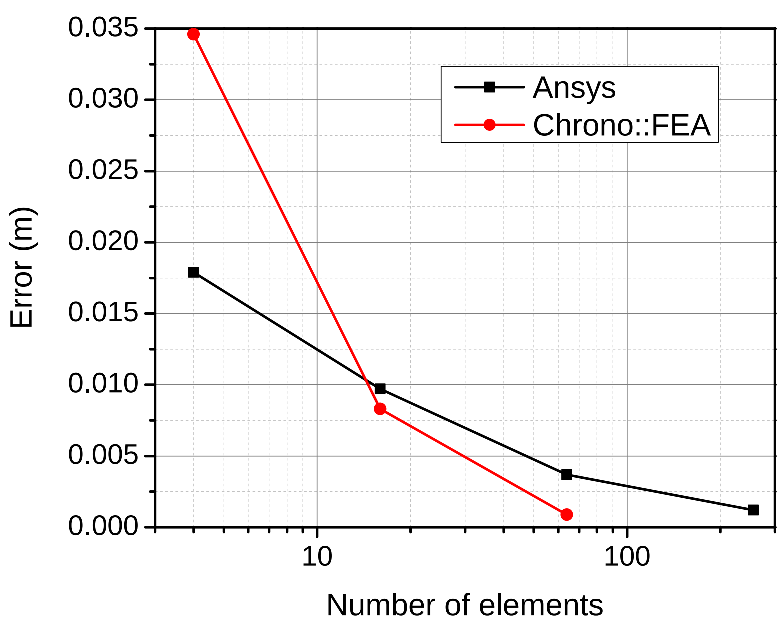

Figure 3: Convergence rate comparison

4.2 Validation of the orthotropic implementation

Chrono allows the user to create laminated shell models using the absolute nodal coordinate formulation. A number of layers with different ply angles and orthotropic elastic constants may be selected. Laminated shell elements have many applications in mechanical engineering problems and, for example, may be used in Chrono to model fully dynamic models of tires.

4.2.1 Model's definition

To validate the laminated shell implementation in Chrono, we have chosen to monitor the response of a quarter of a cylindrical shell. This cylindrical shell is laying in horizontal position with its longitudinal Y axis constrained to the ground at $X = 0$. A load of -10 N is applied at a corner of the free end. Some parameters of the shell geometry are taken as follows: Y length, 1.0 m, single layer thickness, 0.005 m, and cylinder radius, 1.0 m. Elastic constants are chosen in the following way: $E_x = 2.0\text{x}10^8$ Pa, $E_y = E_z = 1.0\text{x}10^8$ Pa, $\nu = 0.3$, and $G_{xy} = G_{yz} = G_{xz} = 3.84615\text{x}10^7$ Pa. The ANCF shell element is chosen to have two stacked layers with ply angles $+20°$ and $-20°$ [1].
This model is defined in Chrono's test_ANCFShellOrt.cpp.

4.2.2 Numerical validation

A sudden force is applied at a free corner of the shell and underdamped motion follows. A reference solution for this problem is obtained in `ANSYS` using a 64x64 mesh of `Shell181` elements (with incompatible modes) and solving a large deformation static analysis: The vertical displacement (not position) for the loaded node is -0.80207 m. The steady state value for that node's displacement is obtained in Chrono by using structural damping to bring the system to equilibrium ($\alpha = 0.25$). When small oscillations are present in Chrono's solution, the mean value of such an oscillation is taken as steady-state response. The time evolution of the loaded tip position for a 6x6 shell is plotted in Fig. 4 (a displacement of approximately -0.8 m may be observed). A convergence study is performed by comparing the steady state of Chrono's simulation results and static solution. The results are shown in Fig. 5, where a displacement of -0.80207 m was taken as a reference.

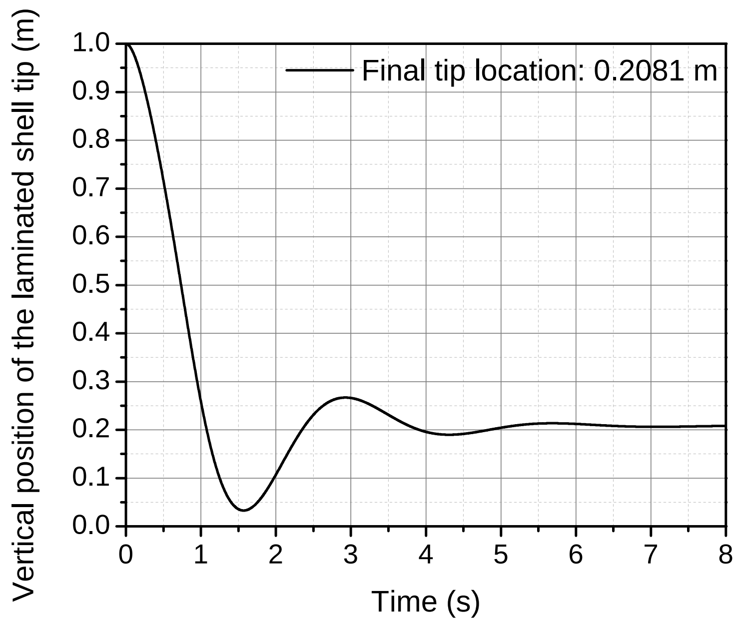

Figure 4: Loaded tip position under the application of sudden load of 10N

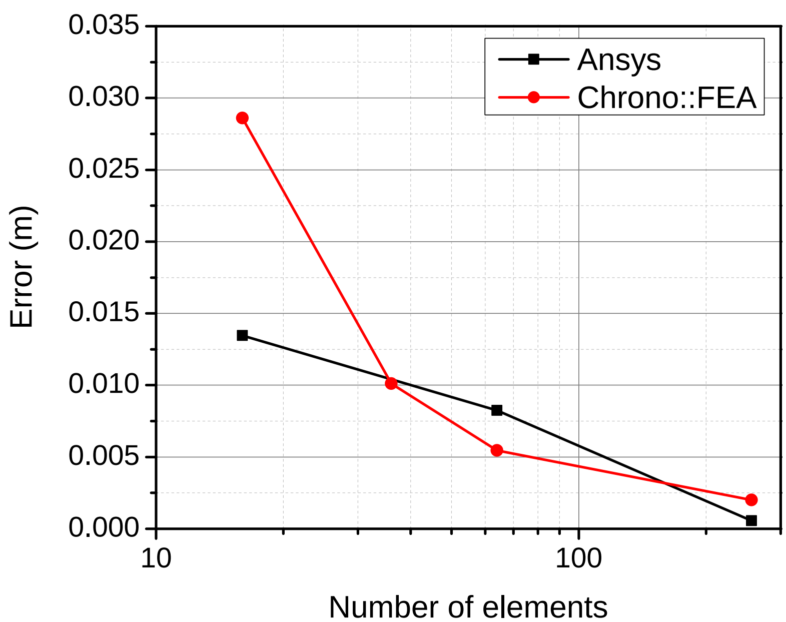

Figure 5: Convergence comparison: `ANSYS` and Chrono

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