Validation of a gradient-deficient ANCF beam element in Chrono::FEA

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Abstract This report describes the verification of the nonlinear finite element implementation of the gradient-deficient ("cable") ANCF element in Chrono. This beam element has one single gradient vector along the beam centerline and may be used for beam applications where shear deformation is negligible. In this document, Chrono's results are compared with the literature in order to verify the the results of gravity, inertia, and internal forces.

Keywords: ANCF beam element, validation, Chrono

Contents

1. Introduction

Chrono's users may find detailed descriptions of the Chrono's ANCF beam implementation in Ref. [2]. Additionally, one can consult the Chrono's website white papers on http://api.chrono.projectchrono.org/whitepaper_root.html.

2. Formulation summary

The ANCF beam element implemented in Chrono is a gradient-deficient element that was introduced by Berzeri and Shabana [1]. This beam element, sometimes called "cable" element, consists of two nodes which have a position vector and a position vector gradient along the beam center axis as coordinates (see Fig. 1). The coordinates of a node k may be expressed as $\boldsymbol{q}^k(t) = \left[\boldsymbol{r}^{k\mathrm{T}} \quad \boldsymbol{r}_x^{k\mathrm{T}}\right]^\mathrm{T}$. The position field of the ANCF beam element is defined as

$$\boldsymbol{r}^i = \left[s_1\boldsymbol{I} \quad s_2\boldsymbol{I} \quad s_3\boldsymbol{I} \quad s_4\boldsymbol{I}\right]\left[\boldsymbol{q}^{1\mathrm{T}} \quad \boldsymbol{q}^{2\mathrm{T}}\right]^\mathrm{T} = \boldsymbol{S}\left(x\right)\boldsymbol{q}^i, \tag{1}$$

where the vector $\boldsymbol{q}^i$ has the coordinates of both nodes, and the shape functions are defined as

$$\begin{aligned} s_1 &= 1 - 2x^2 + 2x^3, \\ s_2 &= l\left(x - 2x^2 + x^3\right), \\ s_3 &= 3x^2 - 2x^3, \\ s_4 &= l\left(-x^2 + x^3\right), \end{aligned} \tag{2}$$

where x is the local parameter of the element ($x = 0$ at the first node) and l is the finite element's reference length.

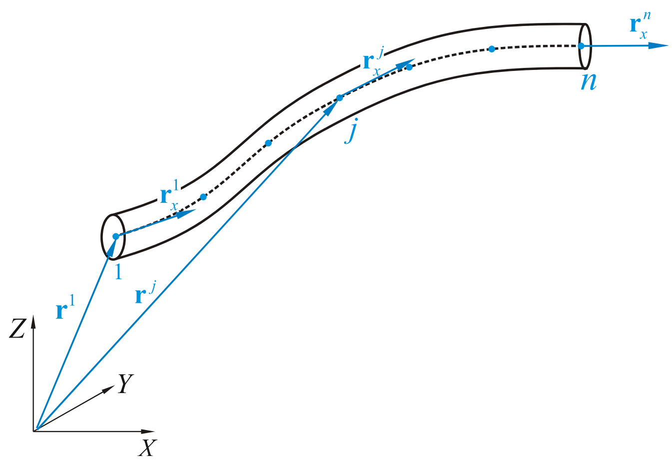

Figure 1: ANCF beam element's kinematic description

Two strains fully define the internal forces of this element: the longitudinal stretch, ε_x, and the curvature, κ. The virtual work exerted by the internal forces may be written as follows

$$\delta W_e = \int_L \left[EA\varepsilon_x\delta\varepsilon_x + EI\kappa\delta\kappa\right]\mathrm{d}x, \tag{3}$$

where E, A, and I are the modulus of elasticity, the cross section area, and the area moment of inertia, respectively; the longitudinal stretch and curvature are

$$\varepsilon_x = \frac{1}{2}\left(\boldsymbol{r}_x^{\mathrm{T}}\boldsymbol{r}_x - 1\right) \text{ and } \kappa = \frac{|\boldsymbol{r}_x \times \boldsymbol{r}_{xx}|}{|\boldsymbol{r}_x|^3},$$

respectively, where $\boldsymbol{r}_{xx} = \partial^2\boldsymbol{r}/\partial x^2$.

3. Validation of the gradient-deficient ANCF beam element

This section shows the results of a validation effort for Chrono's ANCF beam element. The example tested has been extracted from [2] and validates this finite element's elastic force formulation. Additionally, it checks the application of distributed gravity and initial conditions. This validation is performed in Chrono's unit test: test_ANCFBeam.cpp.

3.1 Model's definition

The model used for validating Chrono's implementation is taken from subsection 7.2 of the manuscript [2]. It consists of a beam composed of four finite elements which has one end constrained to the ground through a spherical joint. An initial angular velocity of 4 rad/s about the vertical Y axis is applied by imposing the corresponding initial linear velocity along the X axis. A sketch of the model is shown in Fig.2. The parameters of this model are implemented as in [2], i.e., dependent on a parameter f. Thus, the pendulum length is length 1 m, cross-section area $10^{-6}f^2$ m^2, density $8000/f^2$ kg/m^3, and Young's modulus $10^9/f^4$N/m^2. Poisson's ratio is assumed to be zero and the effect of gravity (z-direction) is considered. The study shown in this report is performed with $f = 5$.

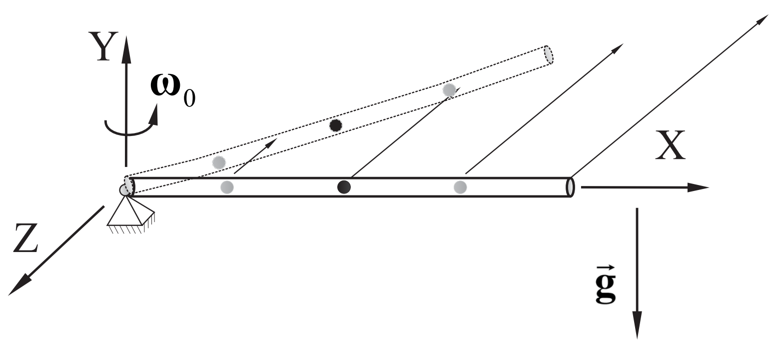

Figure 2: ANCF beam element validation model

3.2 Numerical validation

The three-dimensional pendulum example's results from subsection 7.2 of Ref. [2] were digitized and compare with the results of Chrono's test test_ANCFBeam.cpp, in the Chrono::FEA module. The results may be observed in Figs. 3 and 4. The results show good agreement between published results and Chrono's output. It must be noted that the bending strain formula used in Chrono is that presented in [2] since others have been proposed [3].

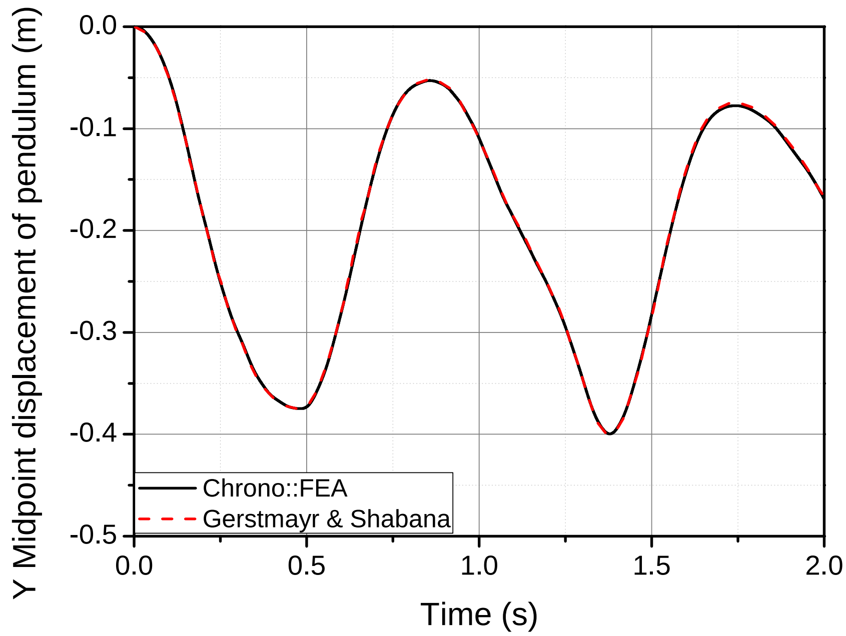

Figure 3: ANCF beam element verification on the vertical displacement

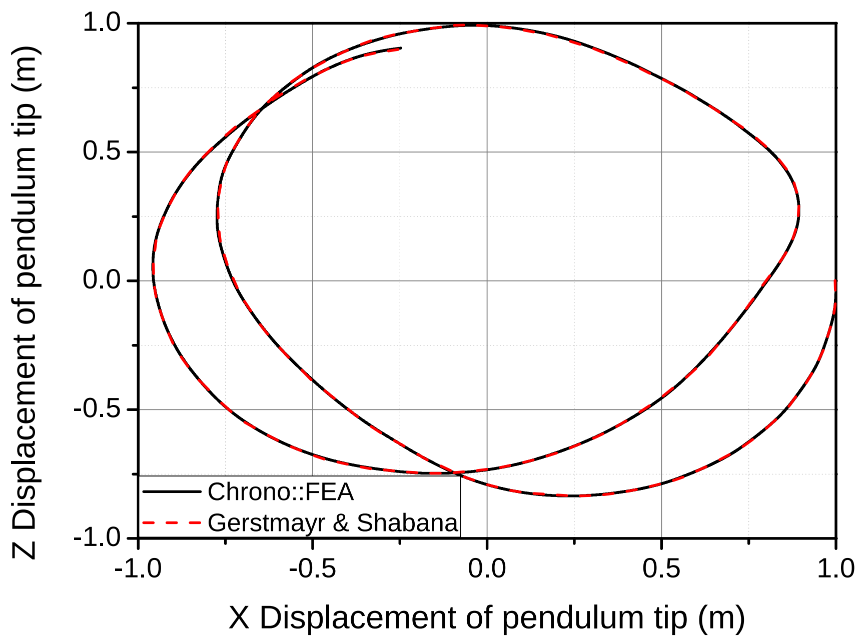

Figure 4: ANCF beam element verification on the horizontal plane

References

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